

Hydro plant modeling

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Water power

$$P = \rho g \eta Q H$$

$$\rho = 1000 \text{ Kg/m}^3$$

$$g = 9.81 \text{ m/sec}^2$$

$$\eta = 0.90 - 0.92$$

$$\mu = 1 \text{ m}^2/\text{sec}$$

Flow

AVERAGE FLOW

River

Parana	17300	cms
Yangtse	15100	cms
Mississippi	17300	cms
Columbia	7500	cms
Snake	1550	cms
Colorado	650	cms
Rio Salado	25	cms

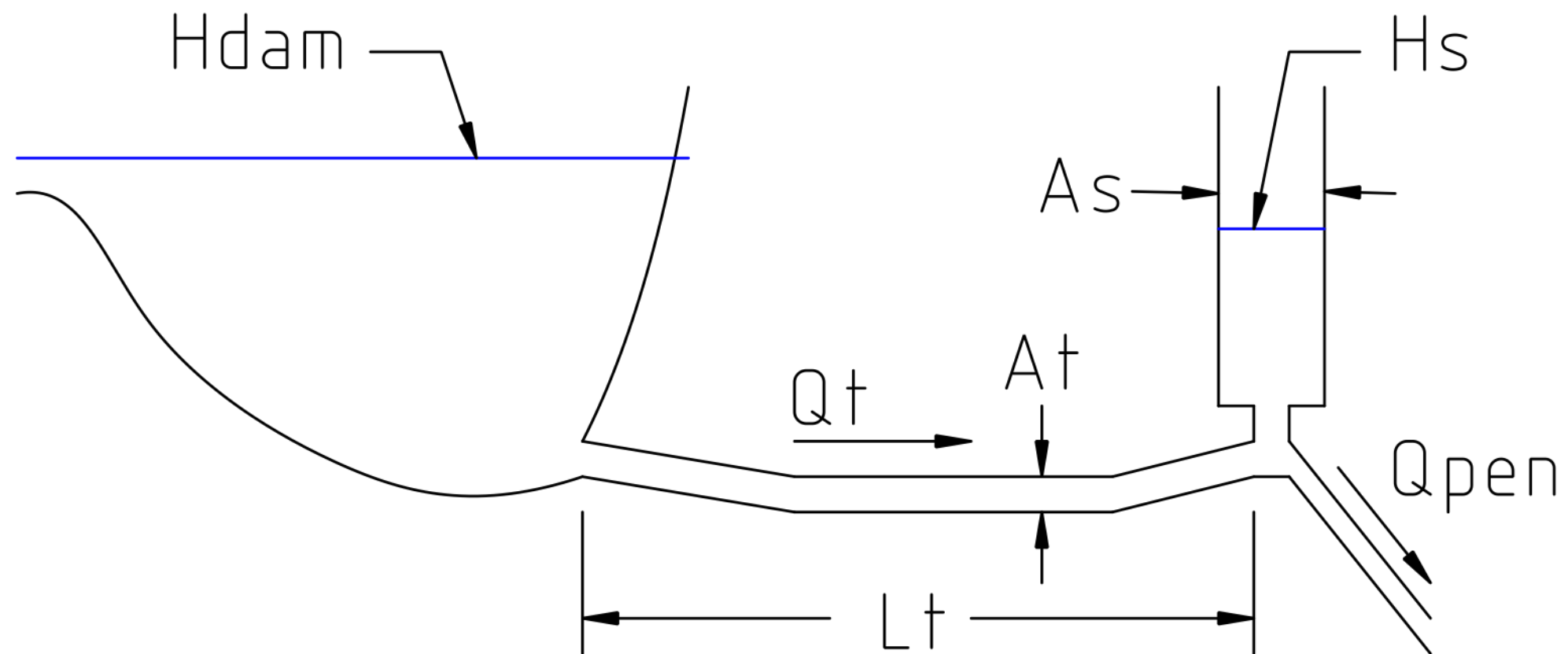
Elevation

MAXIMUM POWER RANGE

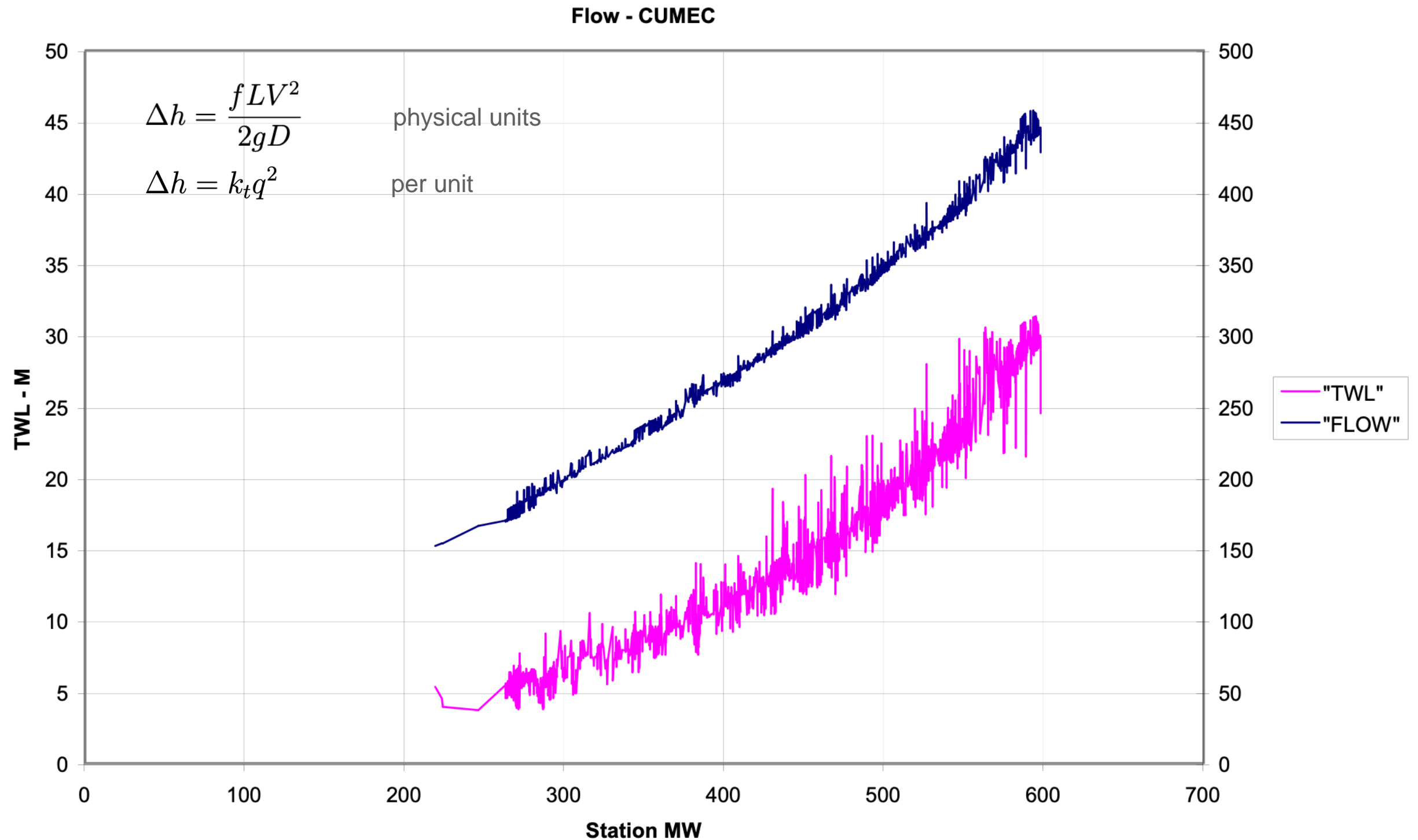
Dam	Minimum Headwater Level	Maximum Level	Rated Head	Power Range
Hoover	950	1220	590	1.75
Grand Coulee	1208	1288	380	1.33
Boundary	1954	1994	340	1.18
Glen Canyon	3524	3624	510	1.31



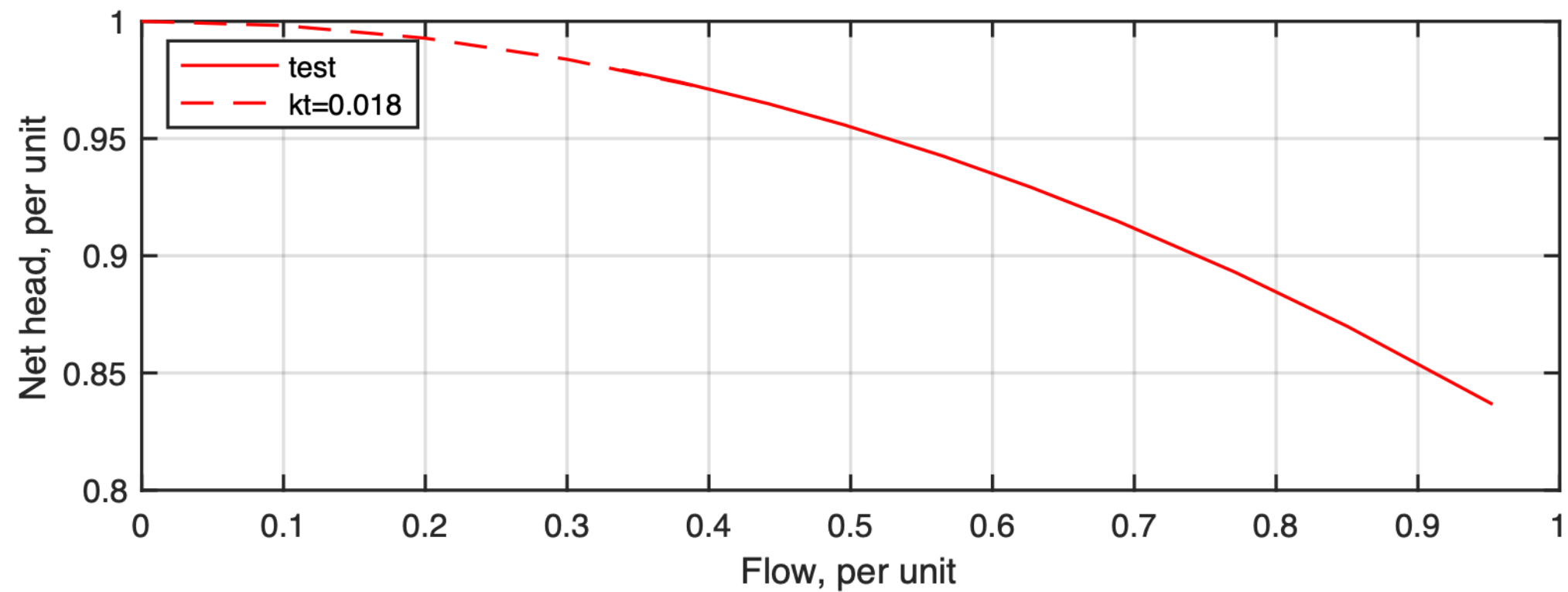
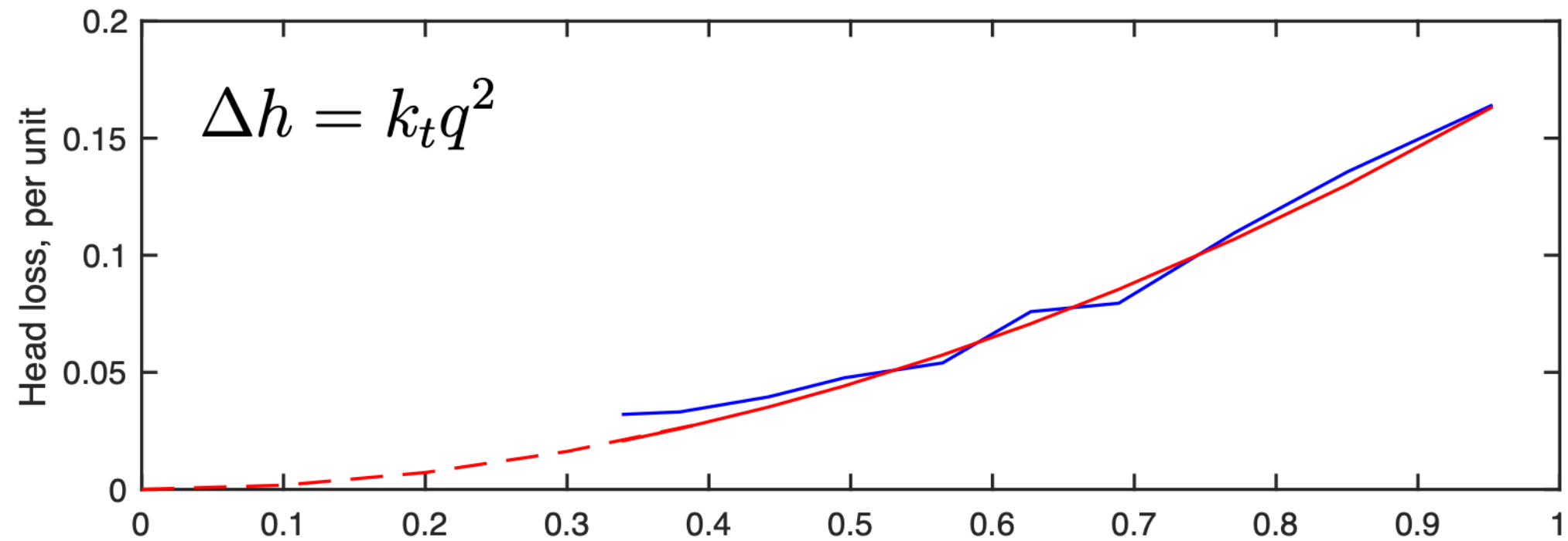
Tunnels and surge chambers

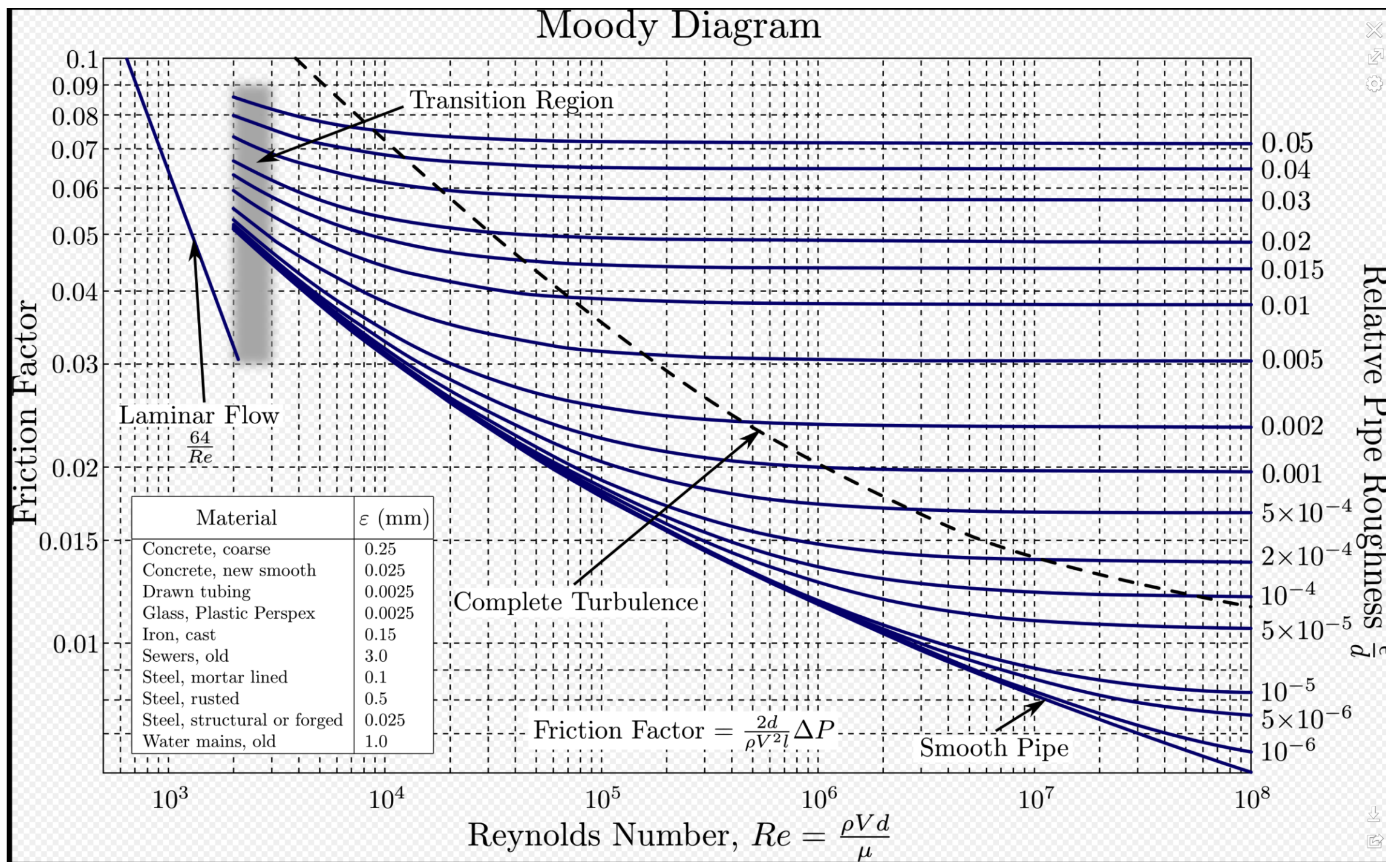


Variation of useful head



Effective head





Moody diagram showing the Darcy–Weisbach friction factor f_D plotted against Reynolds number Re for various relative roughness ϵ / D

[More details](#)

Original diagram: S Beck and R Collins, University of Sheffield ([Donebythesecondlaw](#) at [English Wikipedia](#)) Conversion to SVG: [Marc.derumaux](#) - [File:Moody_diagram.jpg](#)

Moody Diagram

[About Media Viewer](#)

[CC BY-SA 4.0](#)

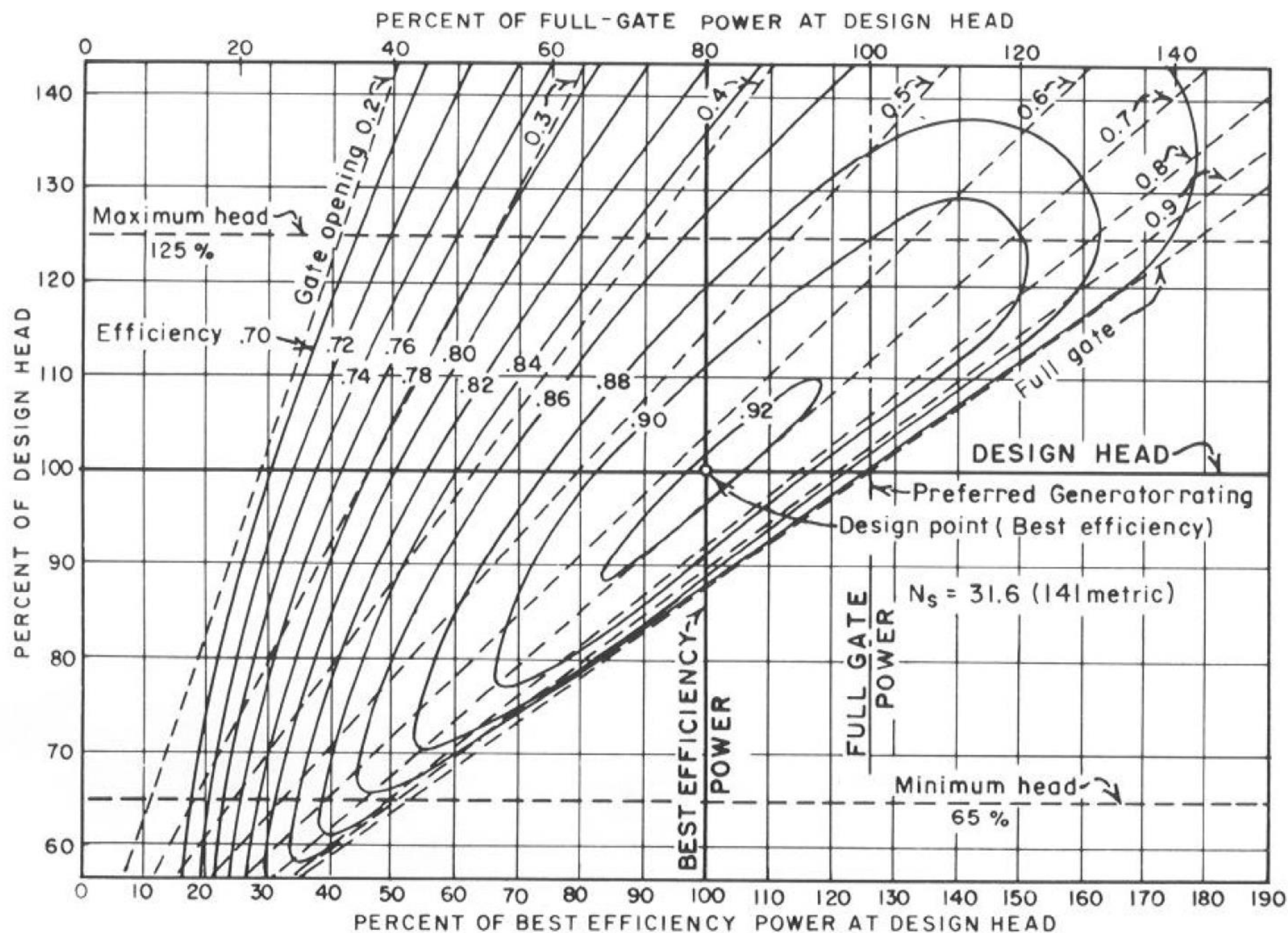
File: [Moody EN.svg](#)

Created: 1 January 2012

Variation of turbine efficiency

106-D-353

FIGURE 9. Typical Francis turbine performance—constant speed and full gate.



■ ■ ■



Turbine characteristic

Part load operation (conceptual turbine modeling)

physical units

$$P = \rho g Q H$$

$$Q = k V \sqrt{H}$$

$$P = \rho g k V H^{1.5}$$

per unit

$$p = q h$$

$$q = v \sqrt{h}$$

$$p = v h^{1.5}$$

$$p = \frac{P}{P_{base}}$$

$$q = \frac{Q}{Q_{base}}$$

$$h = \frac{H}{H_{base}}$$

Part load operation (practical turbine modeling)

per unit

$$p = f_q(q) h$$

$$q = f_v(v) \sqrt{h}$$

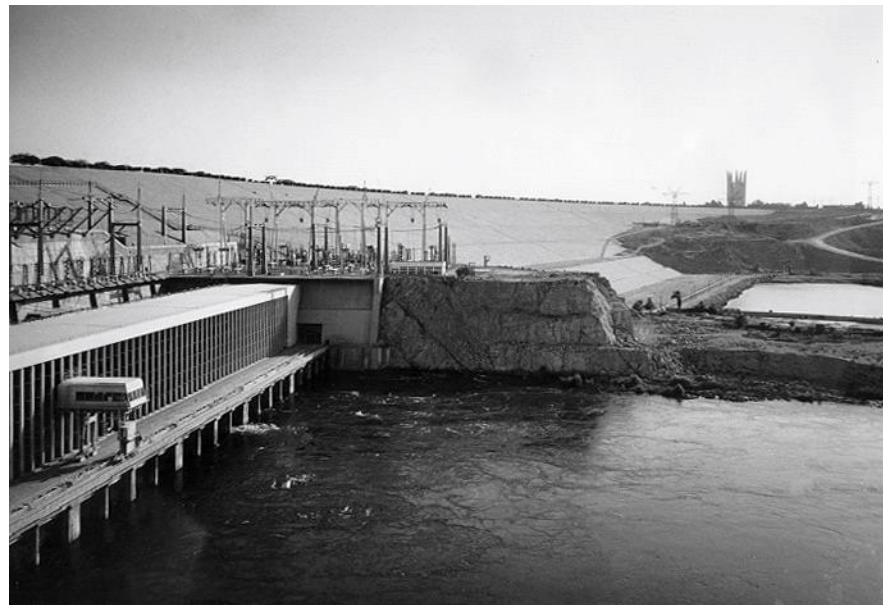
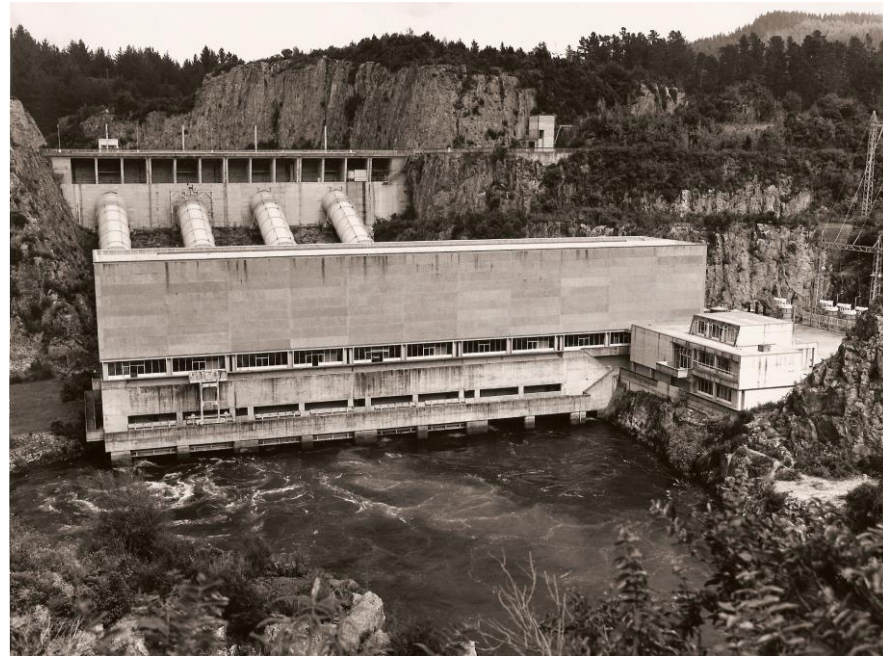
$$p = f(v) h^{1.5}$$

Maximum output $v = 1.0$

$$p = f(1) h^{1.5}$$

$f(1)$ may not be 1.0
River channel can change
and raise tailrace level

Simple hydro plant



Incremental (conceptual) modeling for transfer function

Valve / Flow

Physical units

$$Q = K_t V \sqrt{H}$$

Per unit form

$$q = v \sqrt{h}$$

Incremental form

$$\delta q = h_0 \delta v + v_0 \frac{\delta h}{2}$$

Power

$$P = \rho g Q H$$

$$p = qh$$

$$\delta p = h_0 \delta q + q_0 \delta h$$

$$q_0 = \text{operating flow} \quad 0 < q_0$$

$$h_0 = \text{operating head} \quad 0.8 < h_0 < 1.1$$

Penstock/turbine transfer function

Penstock

Incremental form

$$T_w \frac{dq}{dt} = h - f q^2 \quad T_w = \frac{L_b V_b}{g H_b}$$

Valve

$$\delta q = h_0 \delta v + v_0 \frac{\delta h}{2}$$

Power

$$\delta p = h_0 \delta q + q_0 \delta h$$

Off nominal head – PartLoad

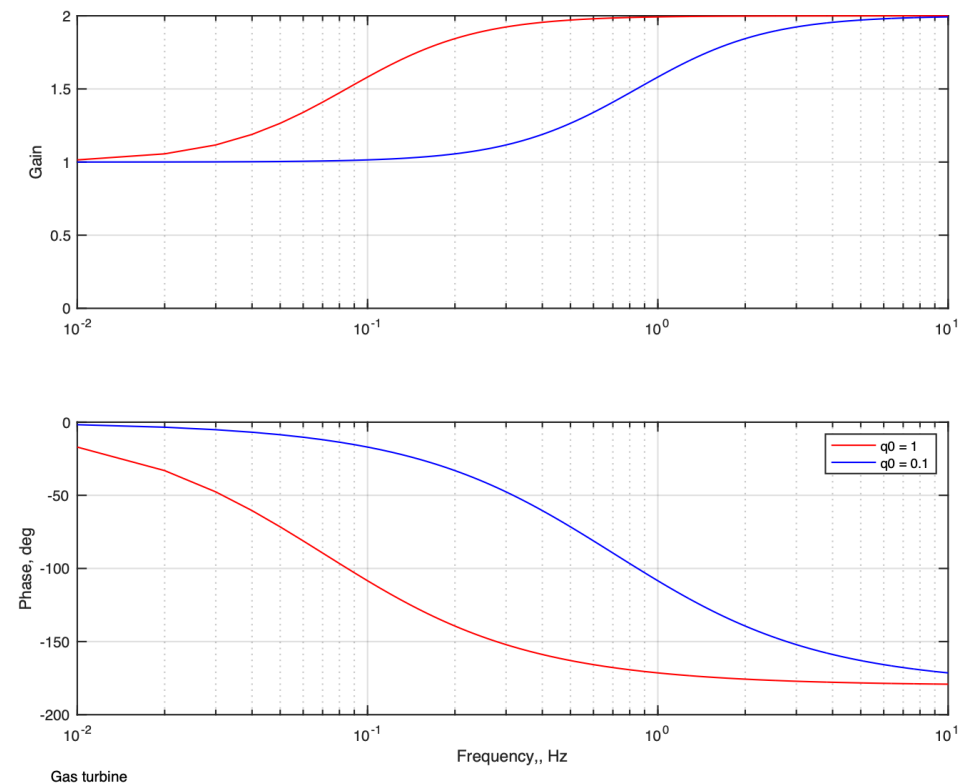
$$\delta p = \left(\frac{1 - \frac{q_0}{h_0} T_w s}{1 + v_0 \frac{T_w}{2} s} \right) \delta v$$

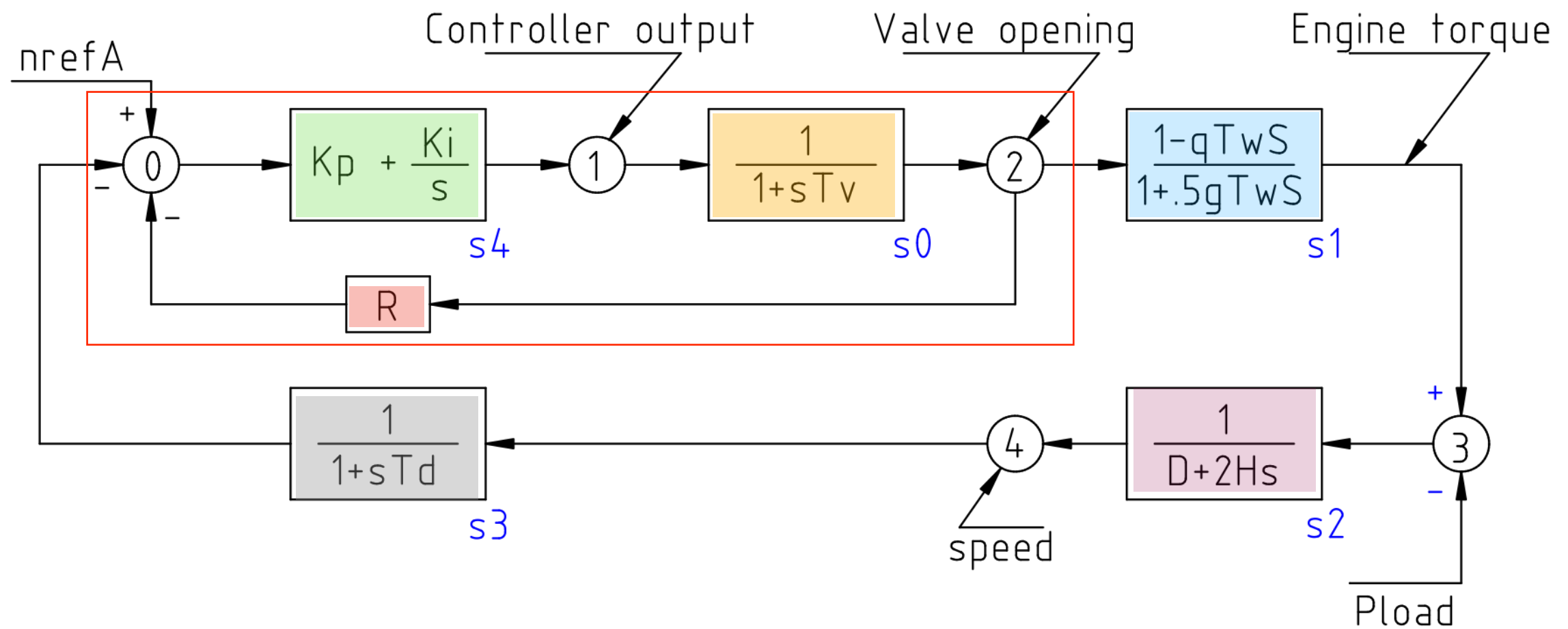
Nominal head – PartLoad

$$\delta p = \left(\frac{1 - q_0 T_w s}{1 + v_0 \frac{T_w}{2} s} \right) \delta v$$

Nominal head – Nominal output

$$\delta p = \left(\frac{1 - T_w s}{1 + \frac{T_w}{2} s} \right) \delta v$$

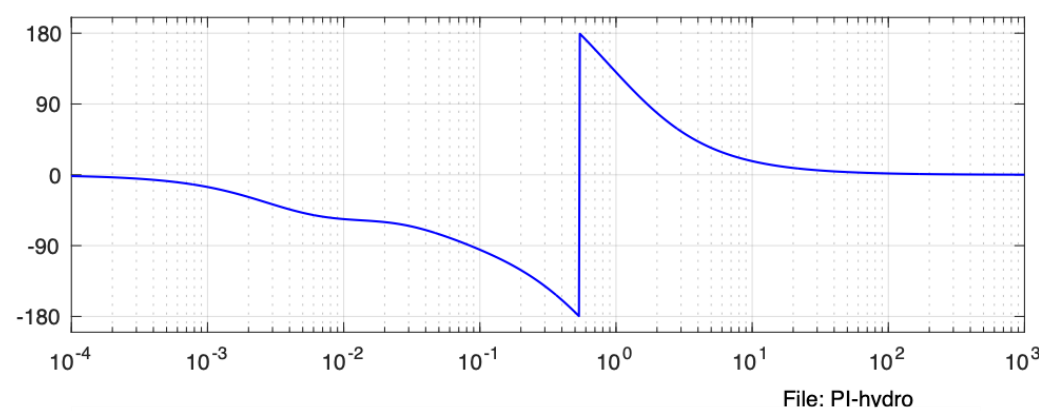
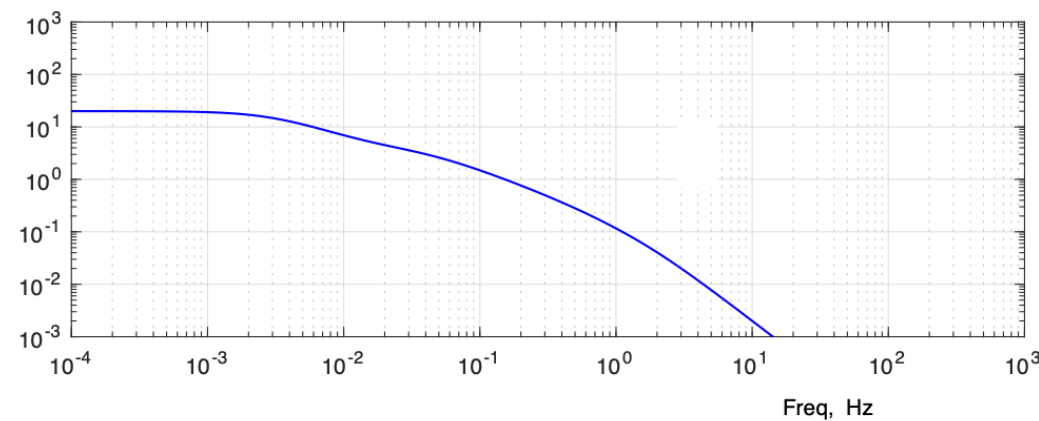
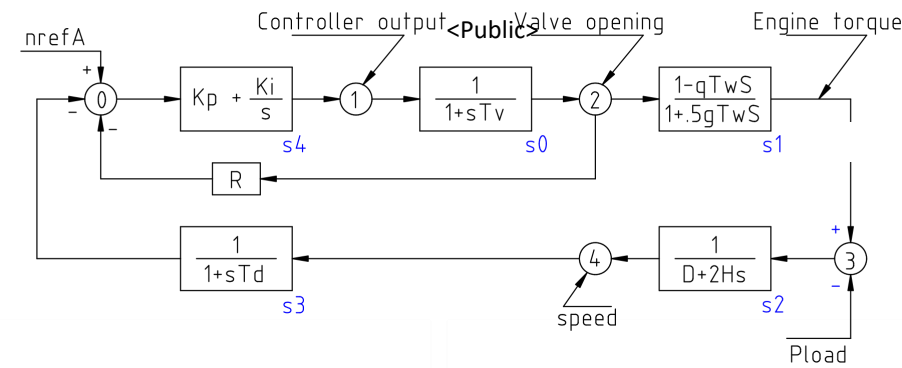




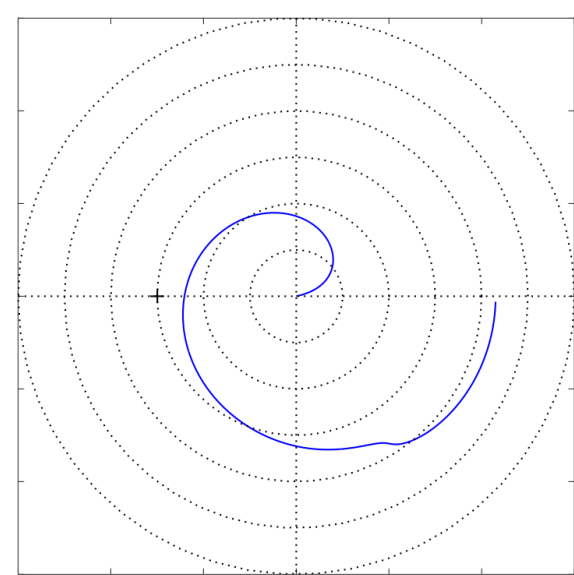
$$G(s) = \frac{1 + sT_r}{R \left(1 + s\frac{r}{R}T_r\right)}$$

$$\delta p = \left(\frac{1 - q_0 T_w s}{1 + v_0 \frac{T_w}{2} s} \right) \delta v$$

$$L(s) = \frac{1}{D + 2Hs}$$

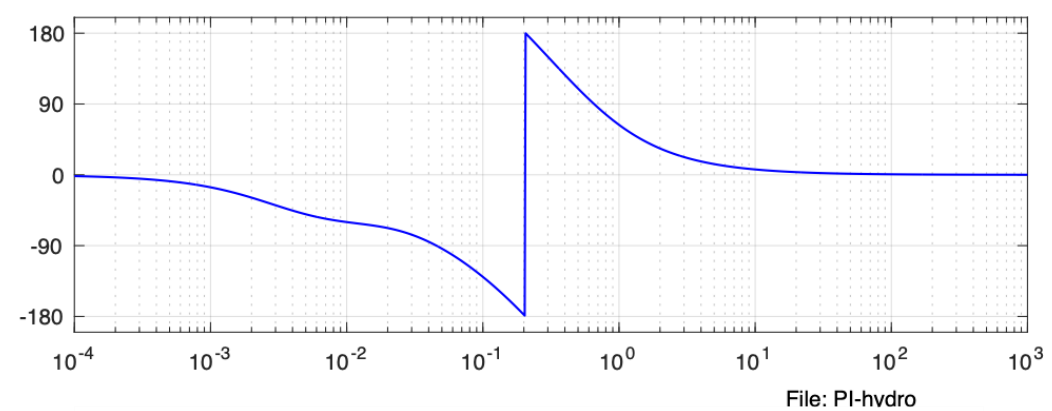
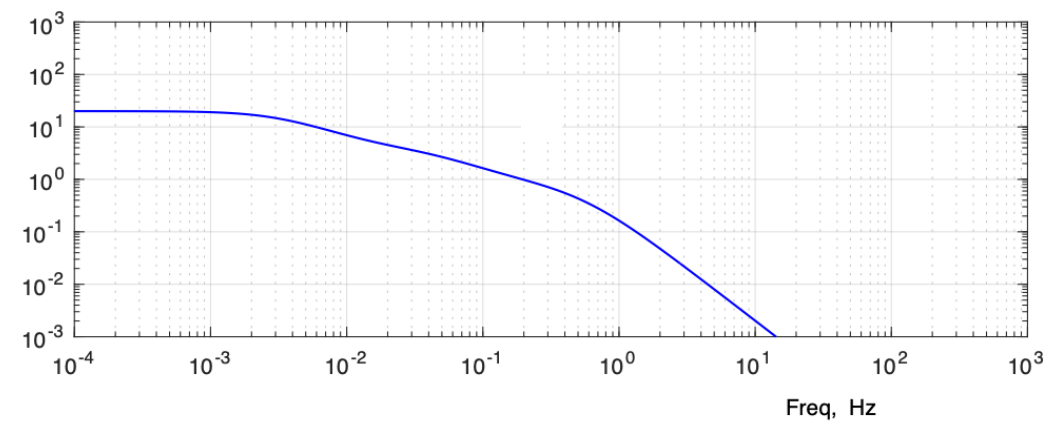


File: PI-hydro

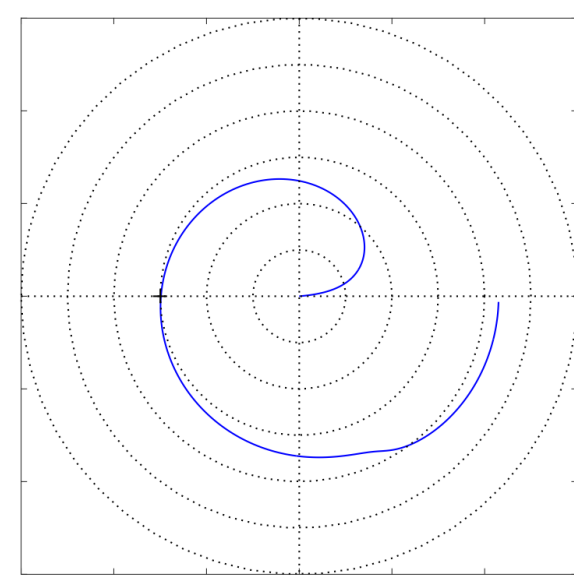


File: PI-hydro

Isolated light load $K_p = 5$



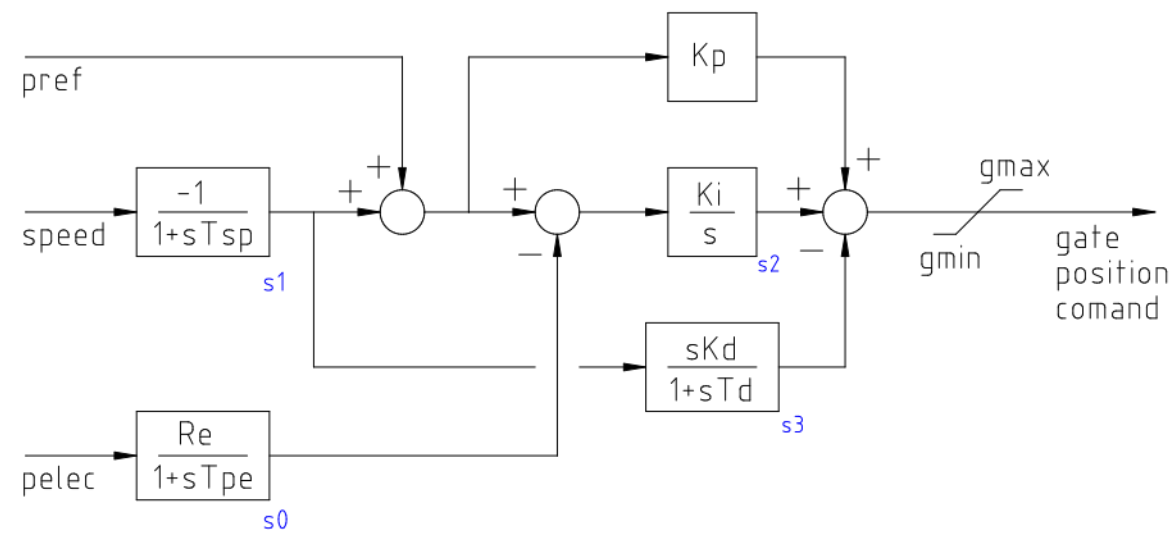
File: PI-hydro



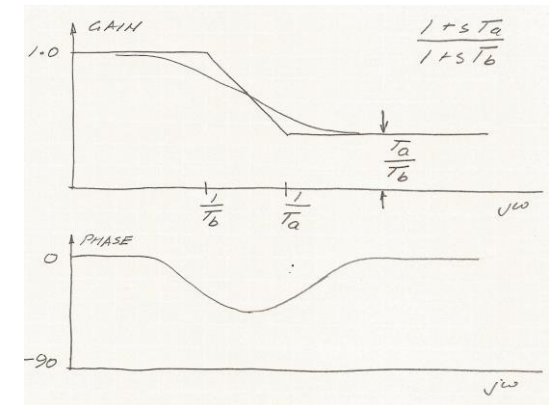
File: PI-hydro

Isolated near full load $K_p = 5$

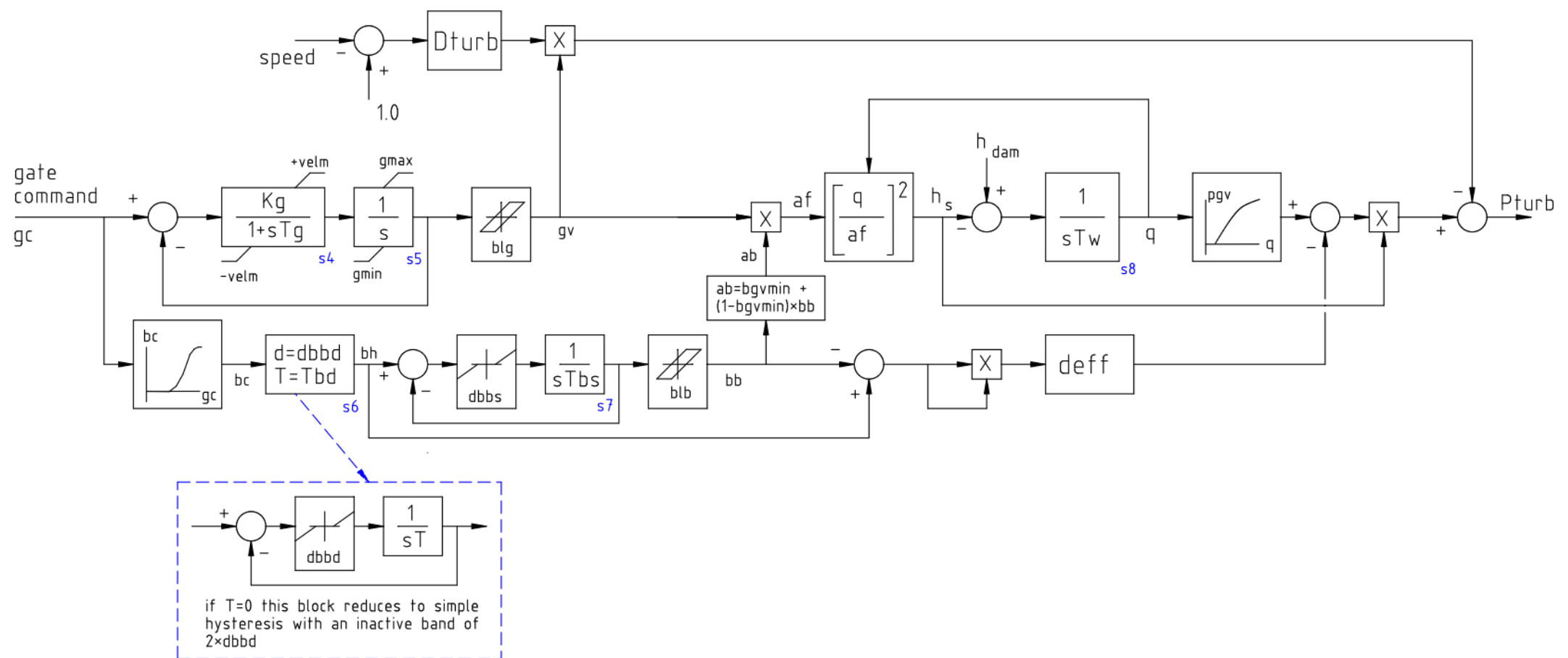
Governor



$$G(s) = \frac{1 + sT_r}{R \left(1 + s\frac{r}{R}T_r\right)}$$

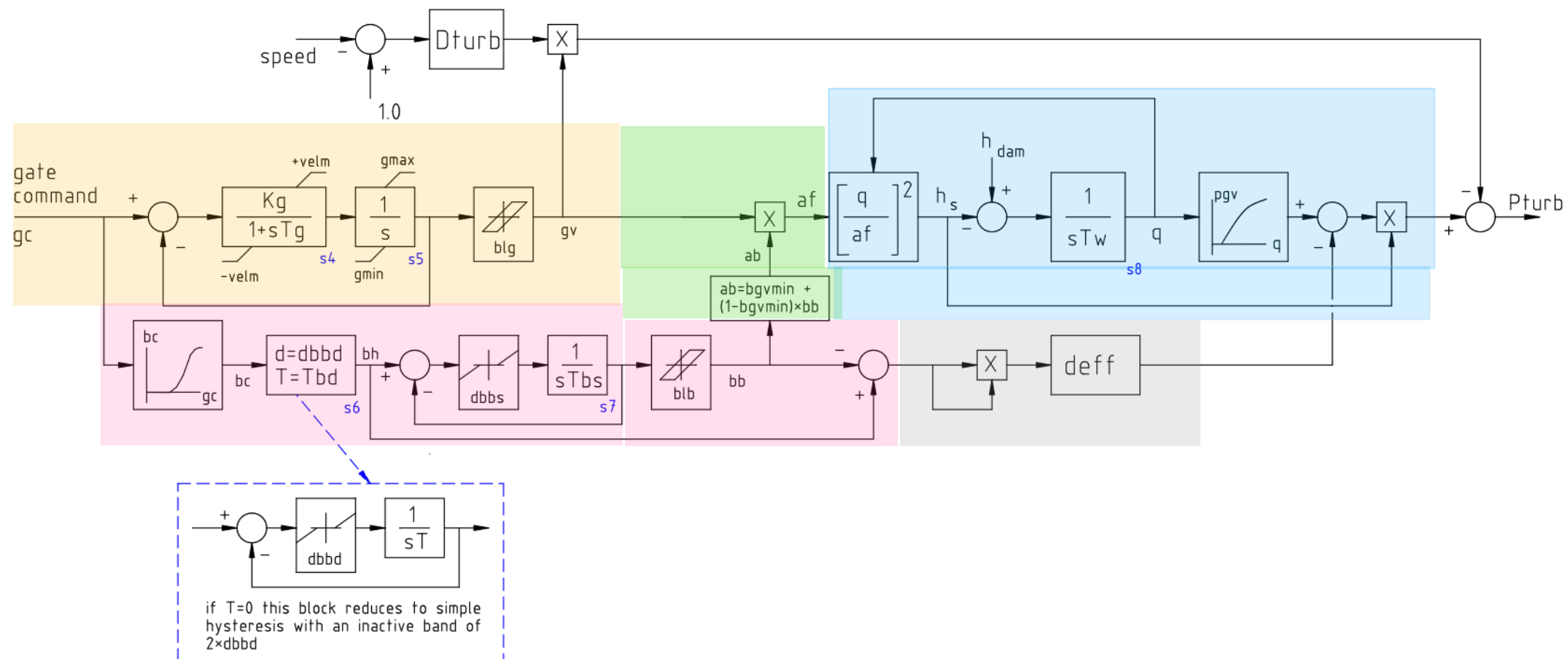
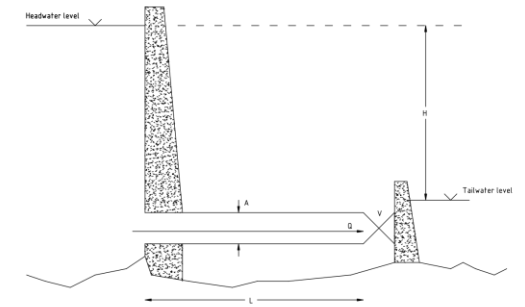


Valve/
Turbine/
Penstock



Turbine/Hydraulic model (simple penstock)

$$\delta p = \left(\frac{1 - \frac{q_0}{h_0} T_w s}{1 + v_0 \frac{T_w}{2}} \right) \delta v \quad T_w = \frac{L_b V_b}{g H_b}$$



Yellow

governor interface

Pink

blade servo

Green

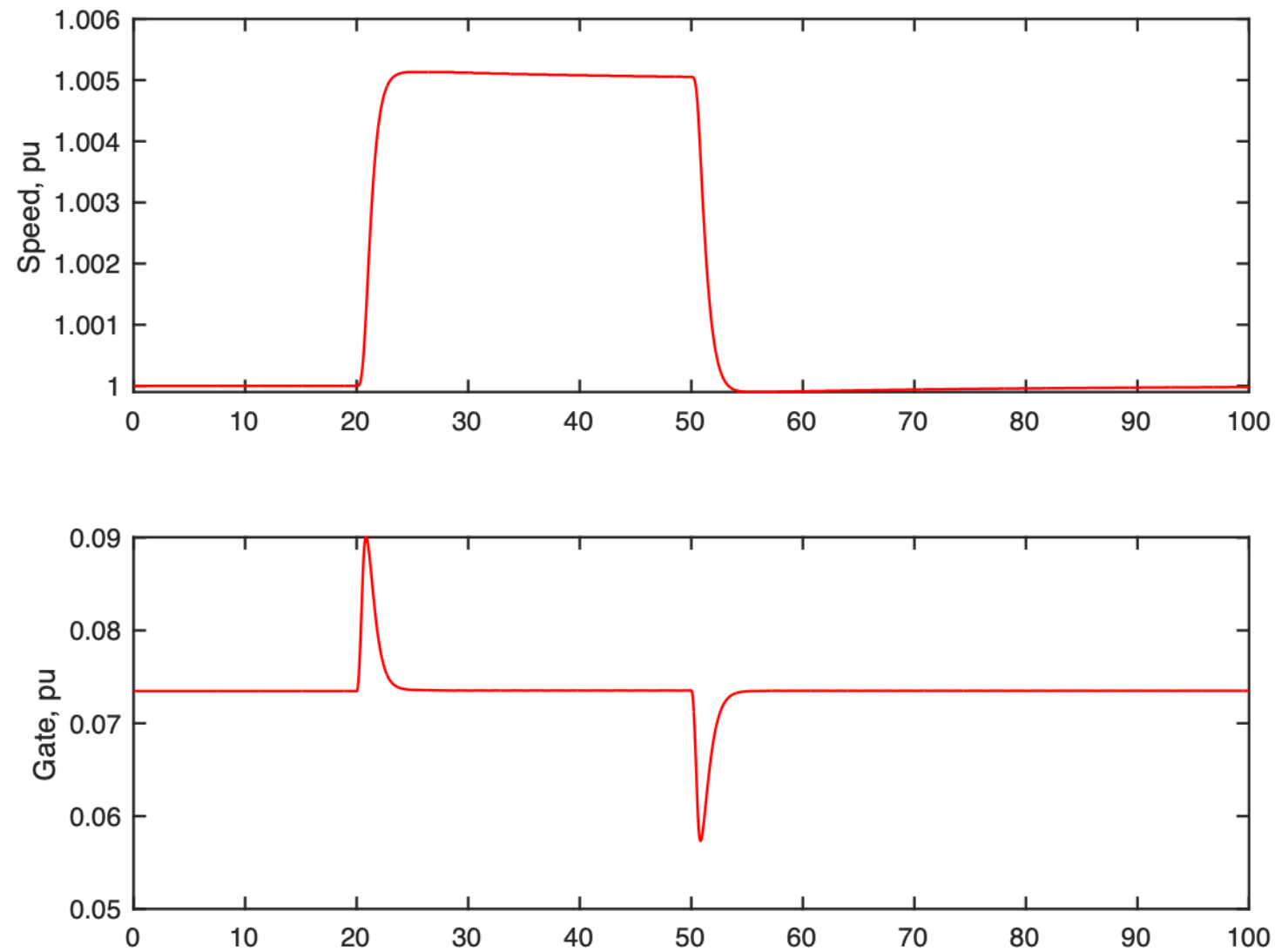
blade angle/flow relationship

Gray

off-cam efficiency allowance

Blue

turbine/penstock model



Off line

Proportional Gain 5.0

Integral gain 0.2

Rotor inertia 3.0

Penstock Tw 2.0

Surge tank Ttank N/A

Tunnel Twt N/A

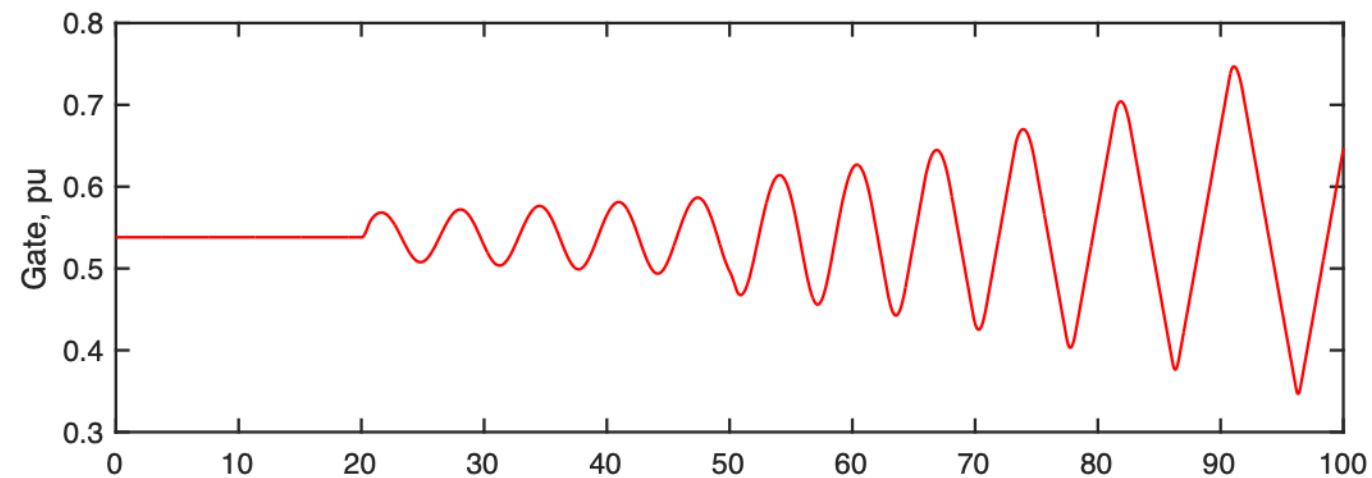
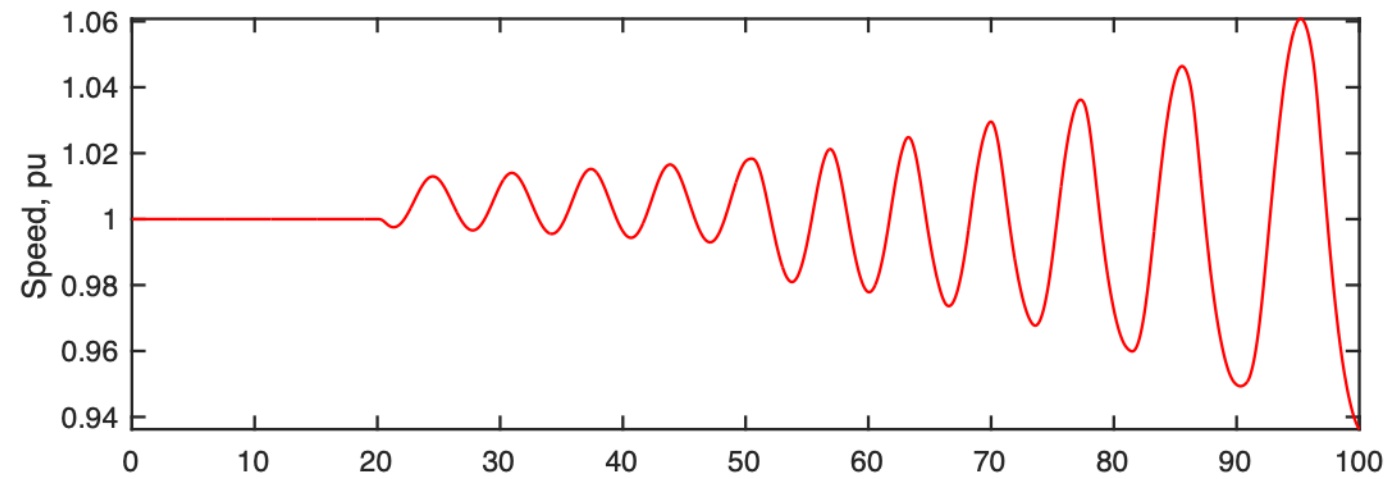
Initial output 0 MW

Load step 0 MW

Governor ref step 0.005%

Generator off line, speed can vary
Governor proportional gain set to give
favorable response to speed adjustment.

Good stability and control when
maneuvering to synchronize



Isolated - loaded

Proportional Gain 5.0

Integral gain 0.2

Rotor inertia 3.0

Penstock Tw 2.0

Surge tank Ttank N/A

Tunnel Twt N/A

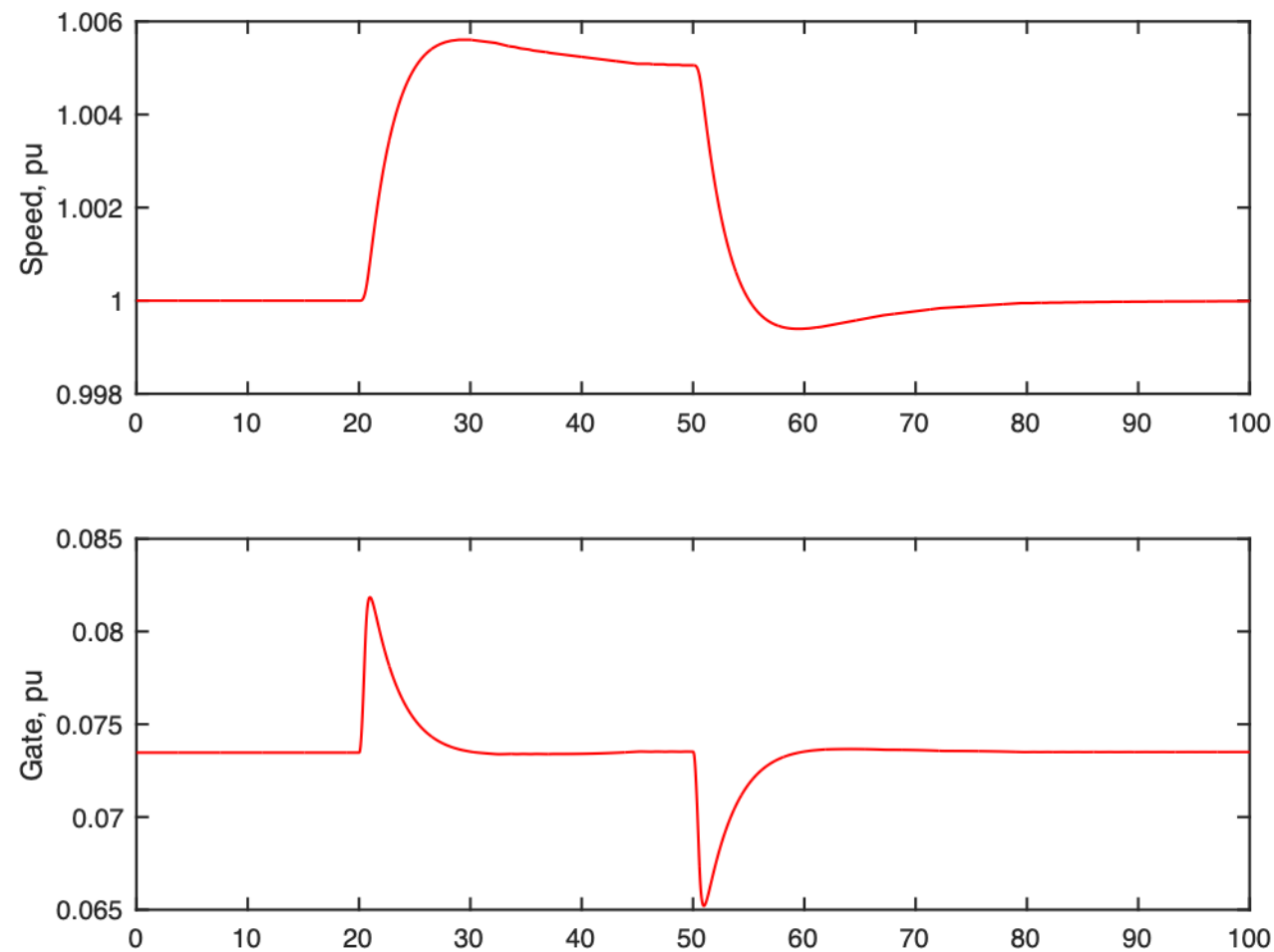
Initial output 40 MW

Load step 0 MW

Governor ref step 0.005%

Generator off line, speed can vary
Governor proportional gain set to give
favorable response to speed adjustment.

Governor gains that gave favorable
behavior at no load are not suitable
for loaded operation in isolation



Off line

Proportional Gain	2.0
Integral gain	0.2
Rotor inertia	3.0
Penstock Tw	2.0

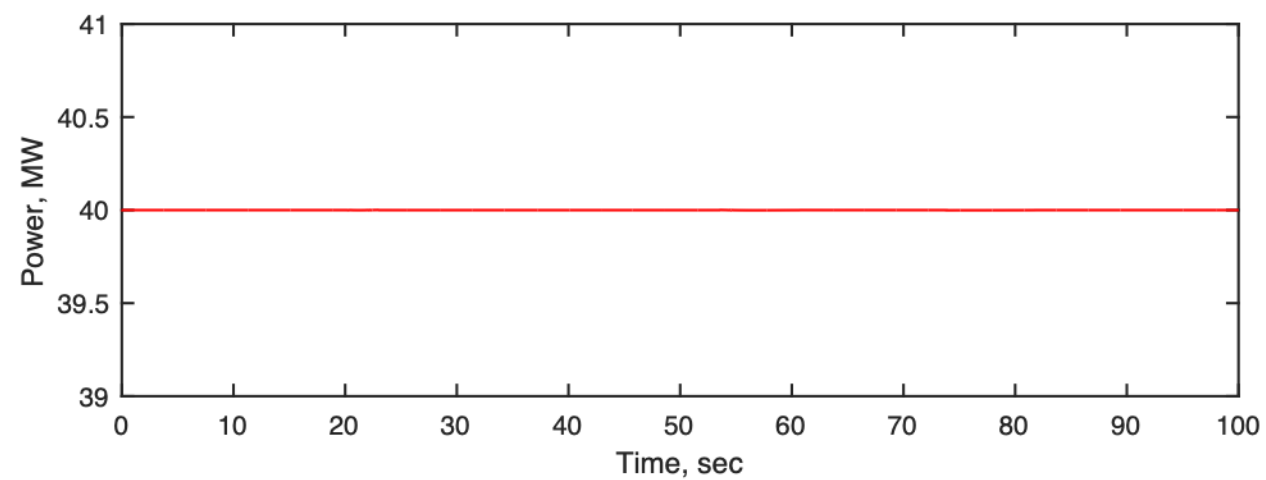
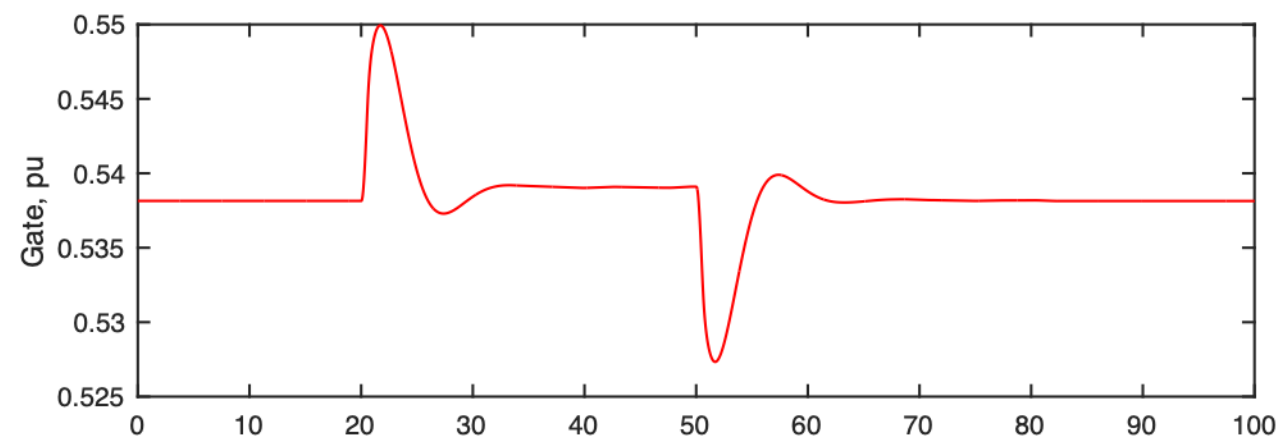
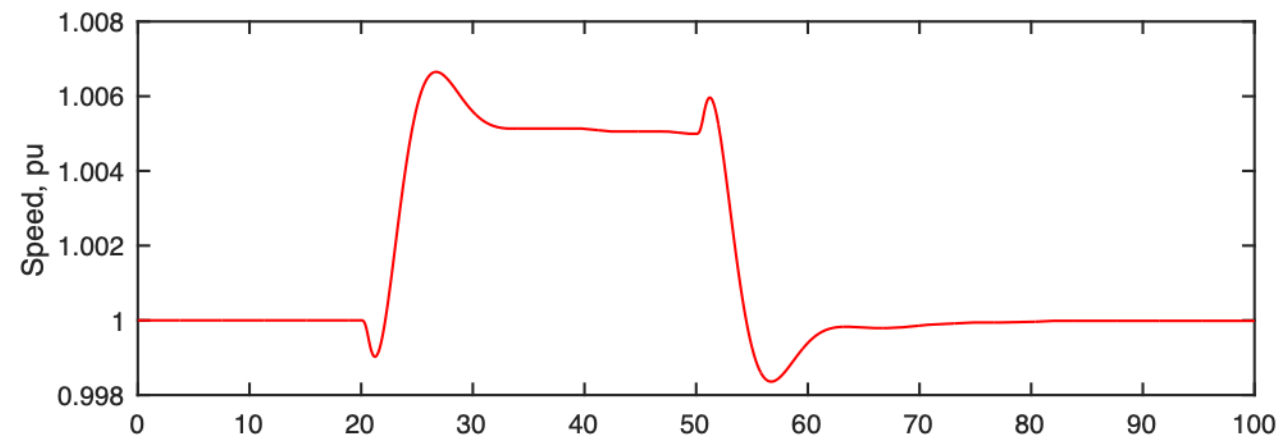
Surge tank T _{tank}	N/A
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Tunnel T _{wt}	N/A
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Initial output	0 MW
Load step	0 MW

Governor ref step	0.005%
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Generator off line, speed can vary
Governor proportional gain reduced
but slows response to speed adjustment.



Isolated - loaded

Proportional Gain 2.0

Integral gain 0.2

Rotor inertia 3.0

Penstock T_w 2.0

Surge tank T_{tank} N/A

Tunnel T_{wt} N/A

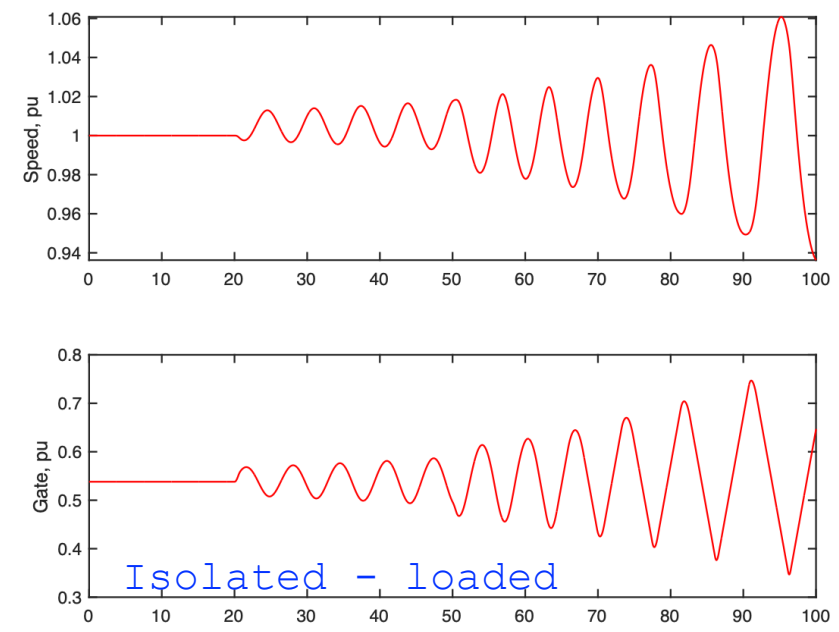
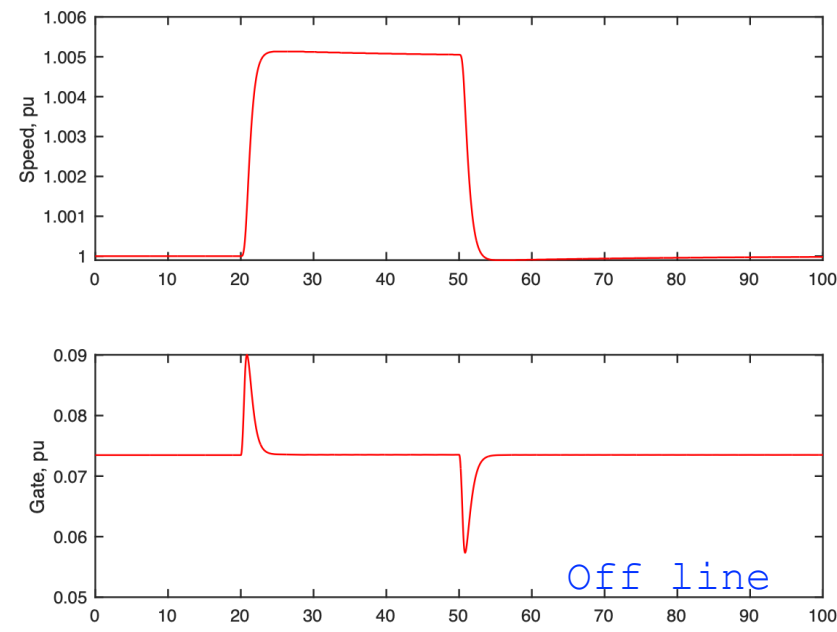
Initial output 40 MW

Load step 0 MW

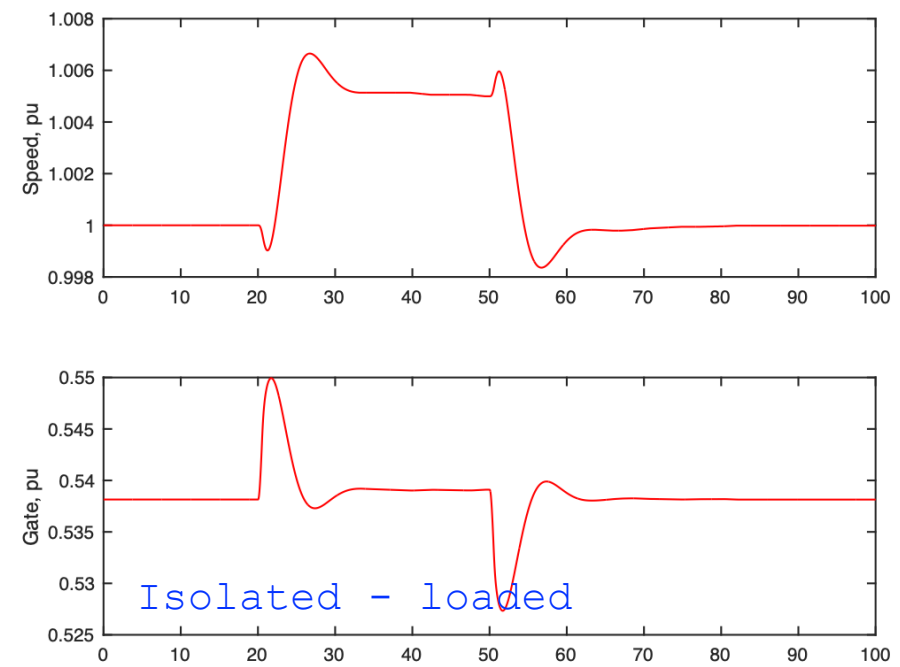
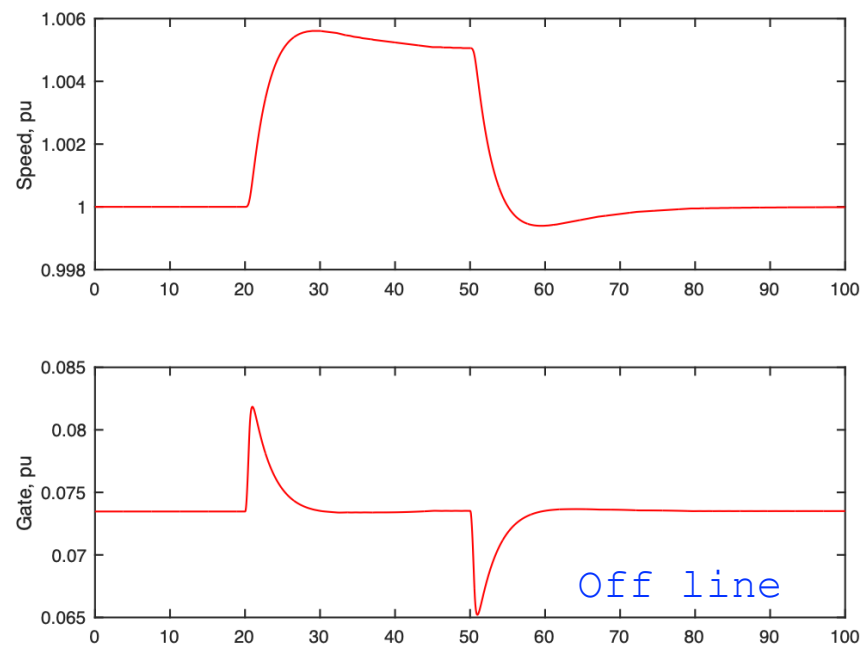
Governor ref step 0.005 %

Generator off line, speed can vary
Governor proportional gain set to give
favorable response to speed adjustment.

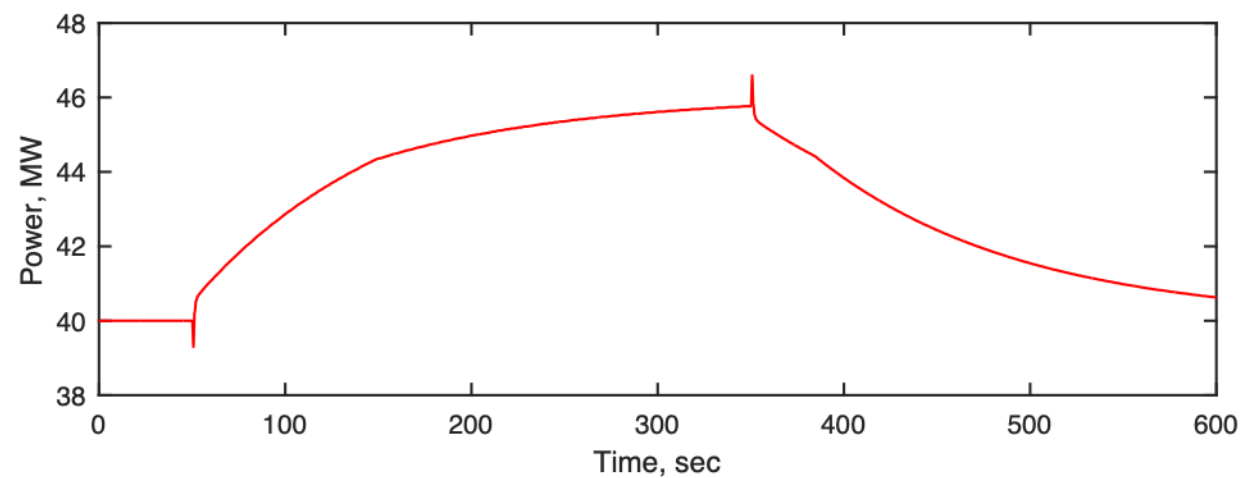
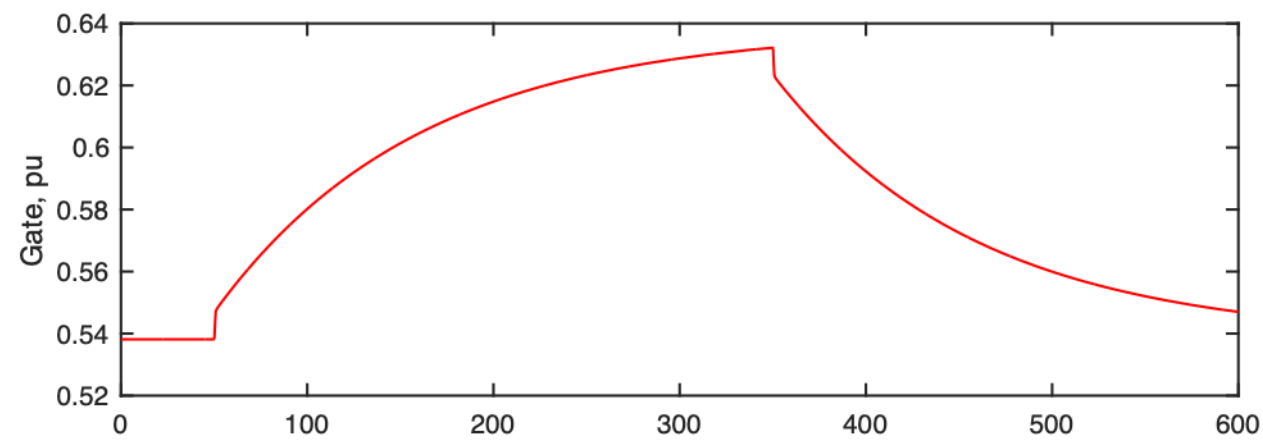
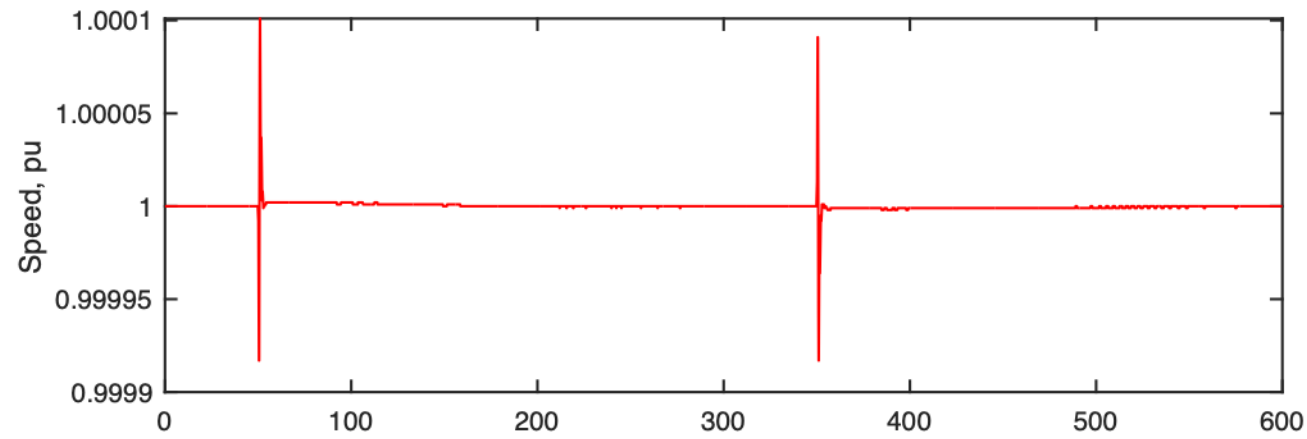
Governor gains that gave stable but slow
response at no load are suitable
for loaded operation in isolation



Governor gain $K_p = 5$



Governor gain $K_p = 2$



On line - loaded

Proportional Gain 2.0

Integral gain 0.2

Rotor inertia 3.0

Penstock Tw 2.0

Surge tank T_{tank} N/A

Tunnel T_{wt} N/A

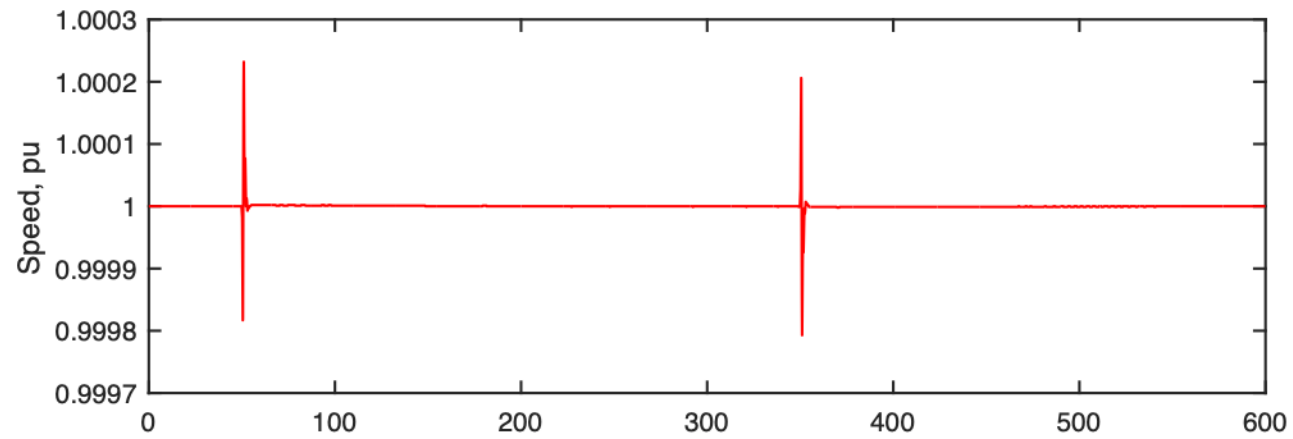
Initial output 40 MW

Load step 0 MW

Governor ref step 0.005 %

Generator on line, speed substantially fixed.

Governor gains that gave stable but slow response at no load are safe for loaded operation connected to a strong grid



On line - loaded

Proportional Gain 5.0
 Integral gain 0.2
 Rotor inertia 3.0
 Penstock Tw 2.0

Surge tank T_{tank} N/A

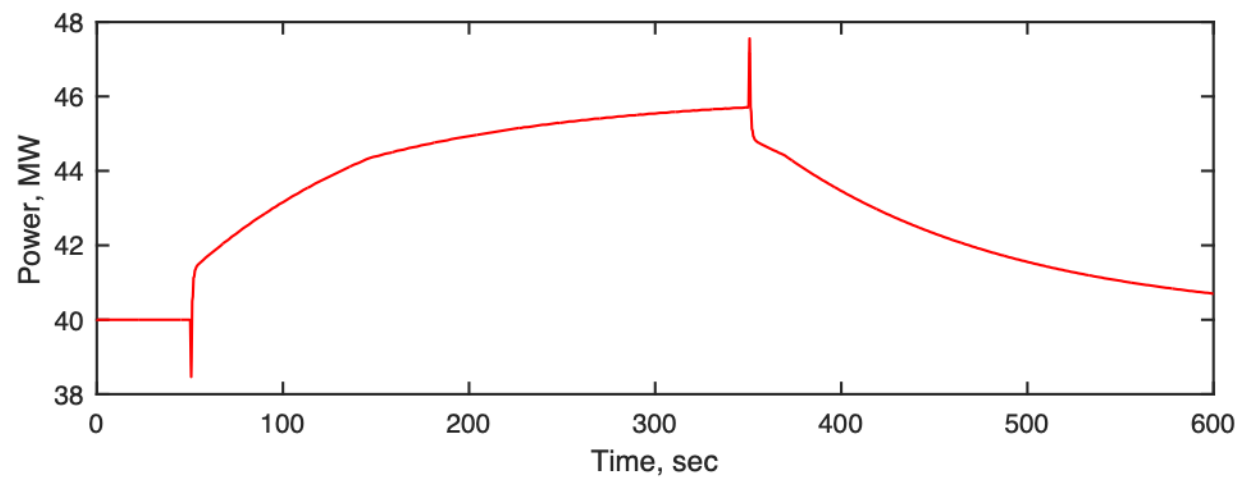
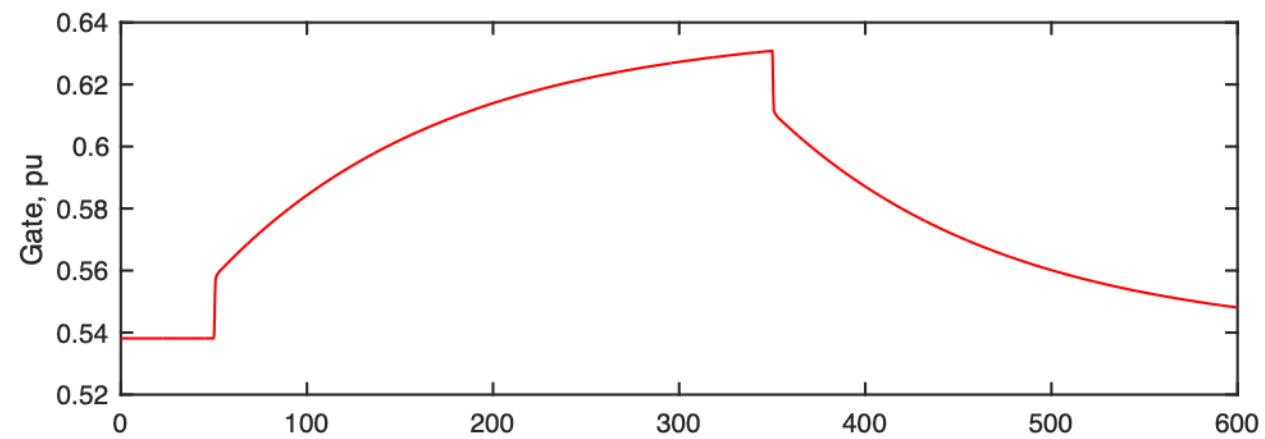
Tunnel T_{wt} N/A

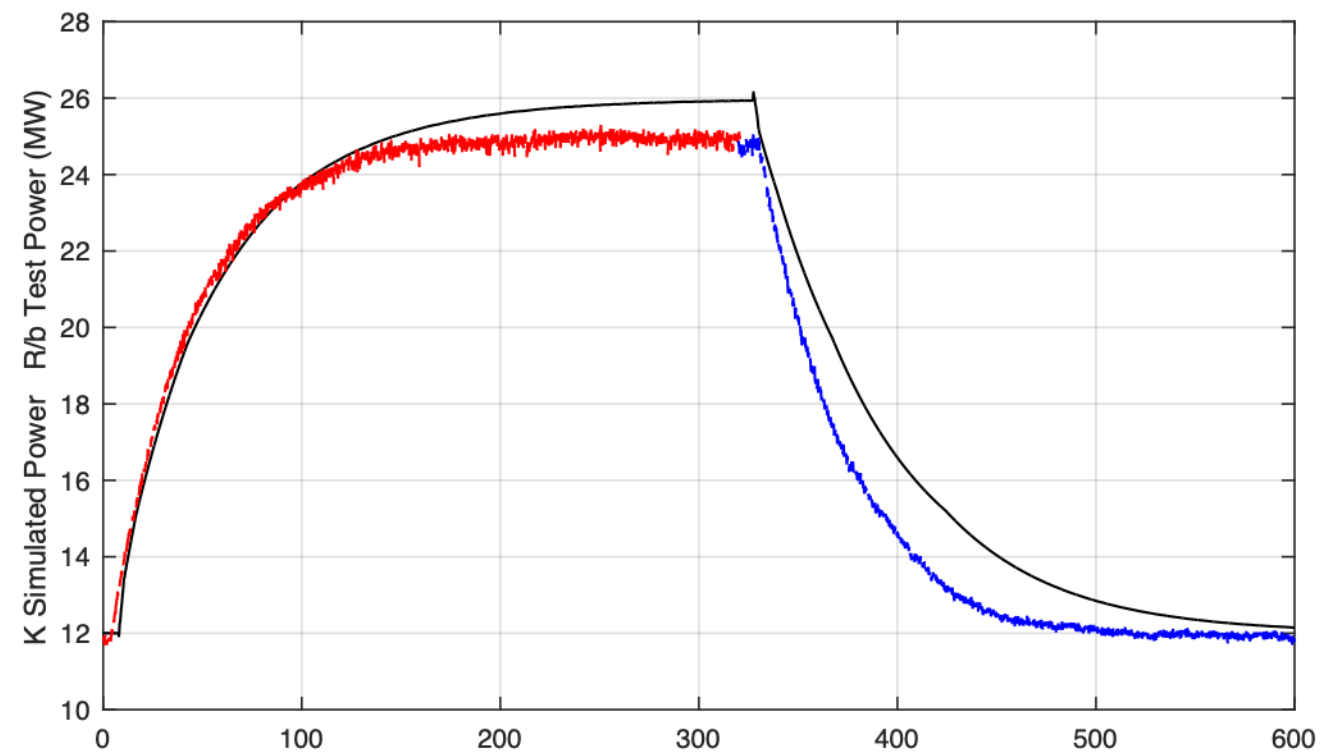
Initial output 40 MW
 Load step 0 MW

Governor ref step 0.005 %

Generator on line, speed substantially fixed
 Governor proportional gain set to give favorable response to speed adjustment.

Governor gains that gave stable favorable speed response when isolated at no load give satisfactory power response when connected to a strong grid

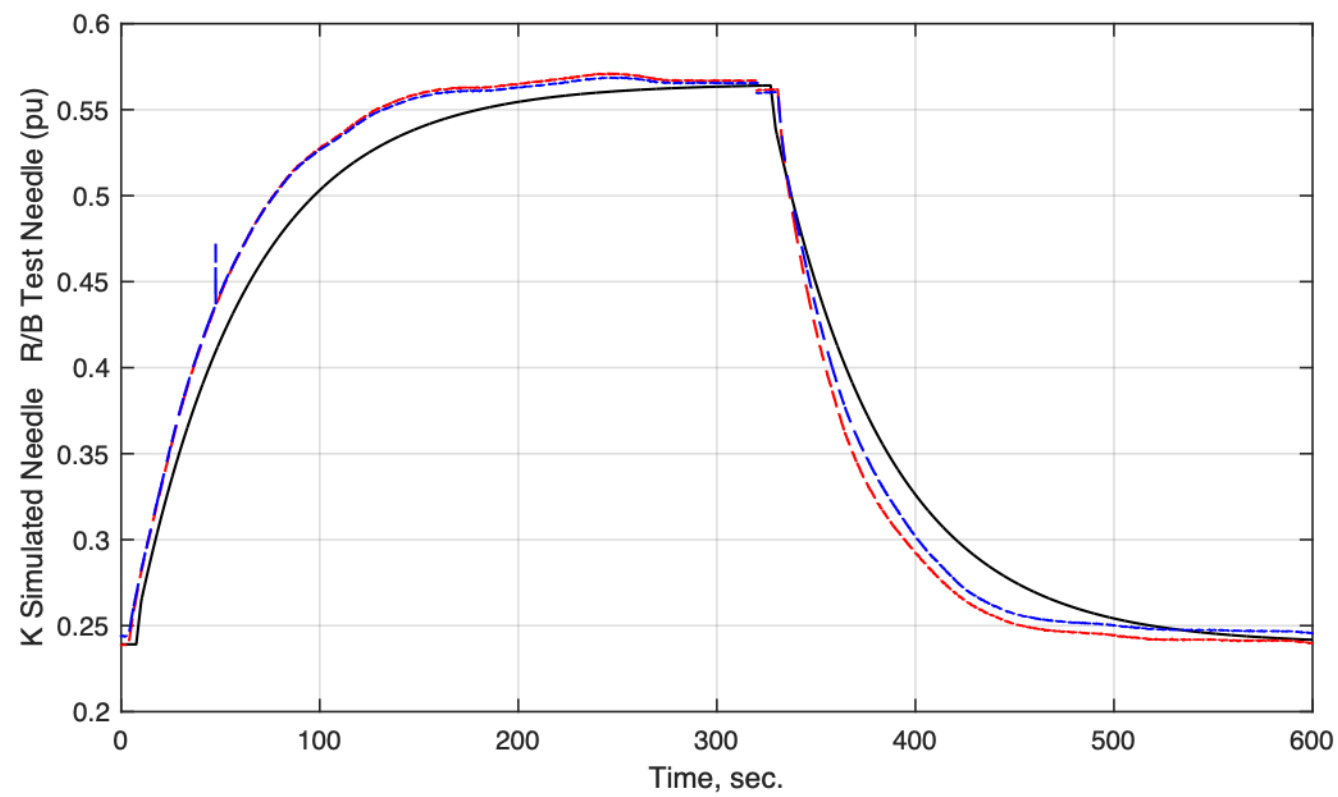




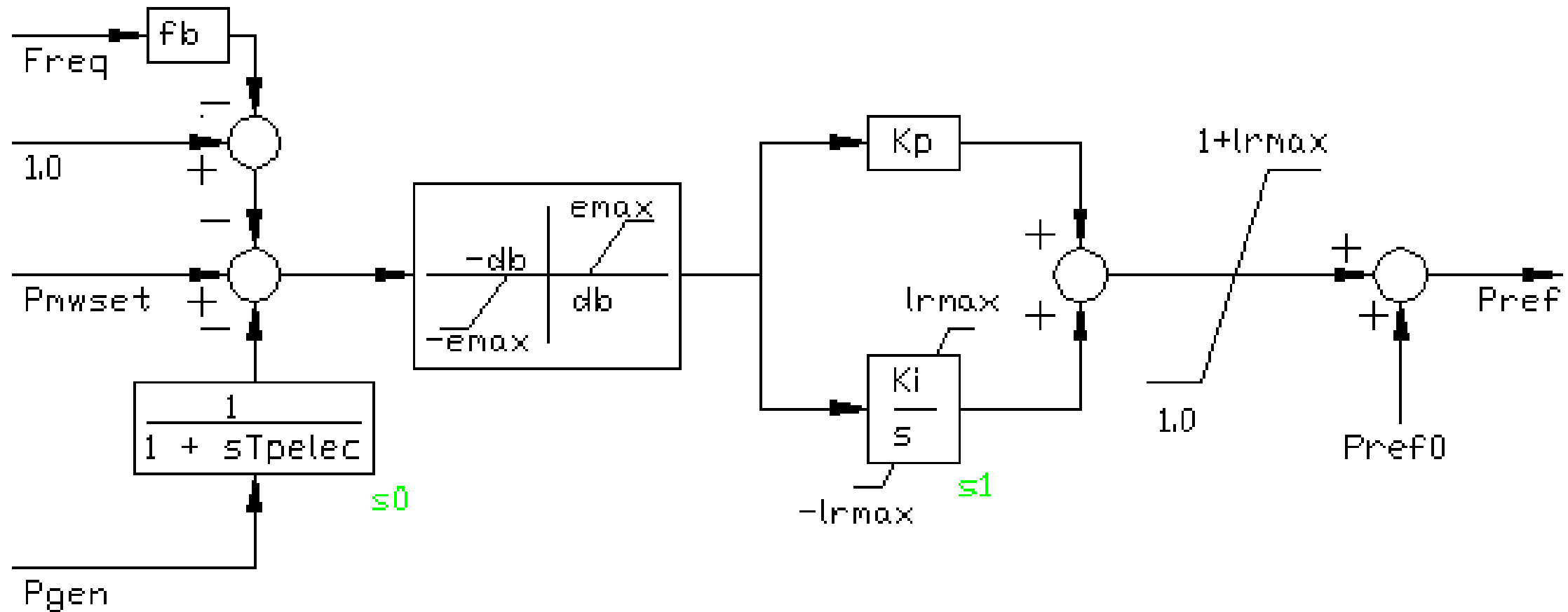
On line

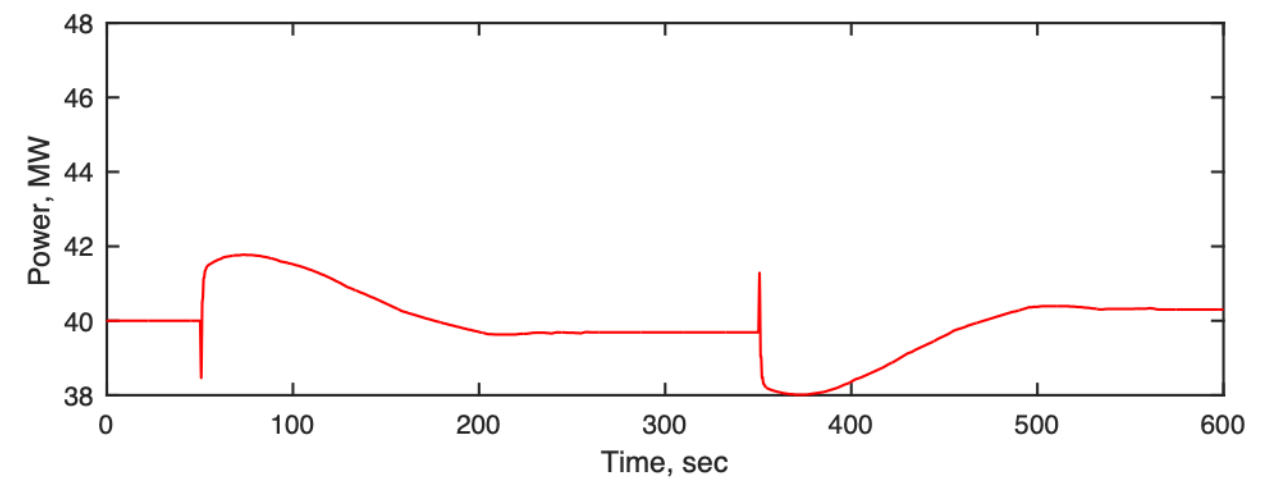
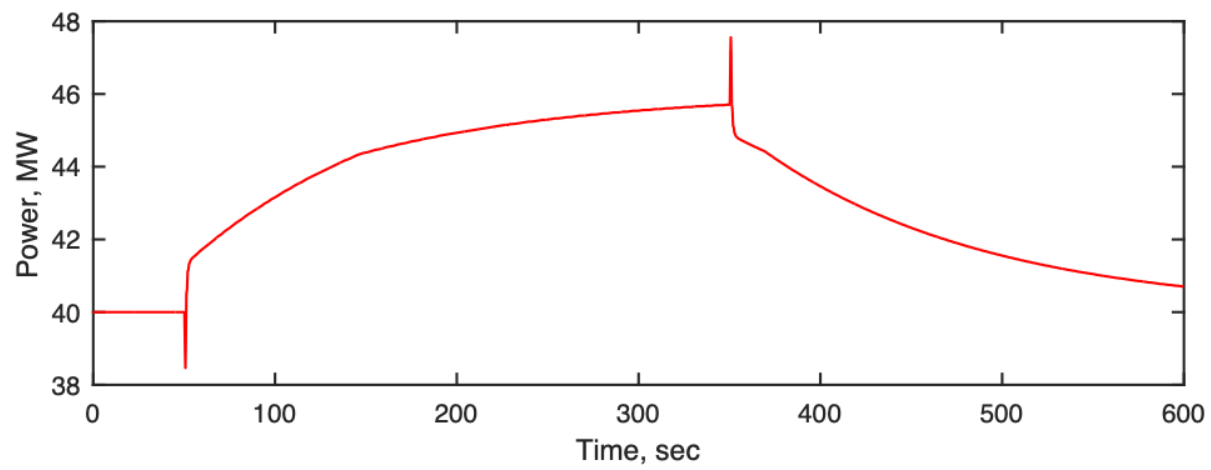
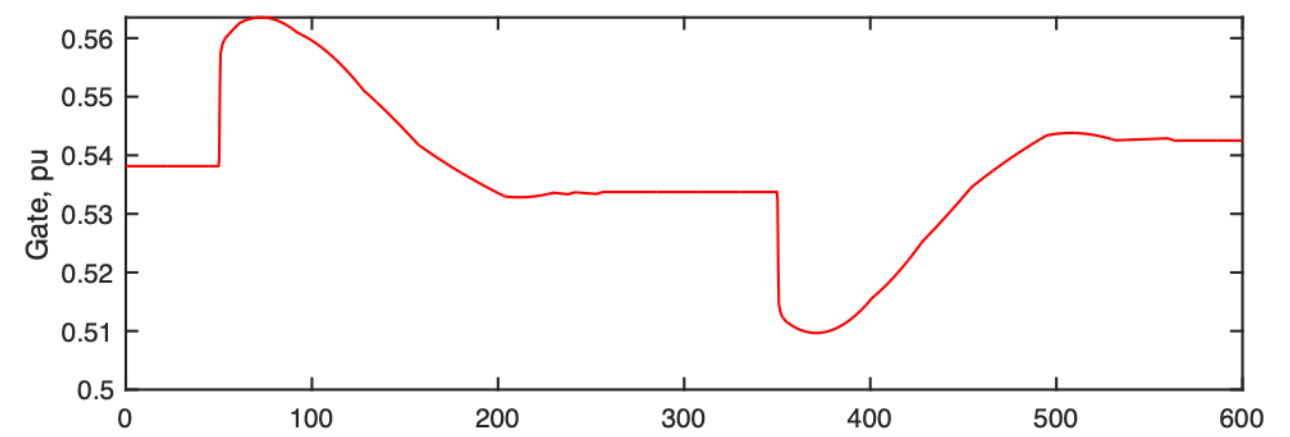
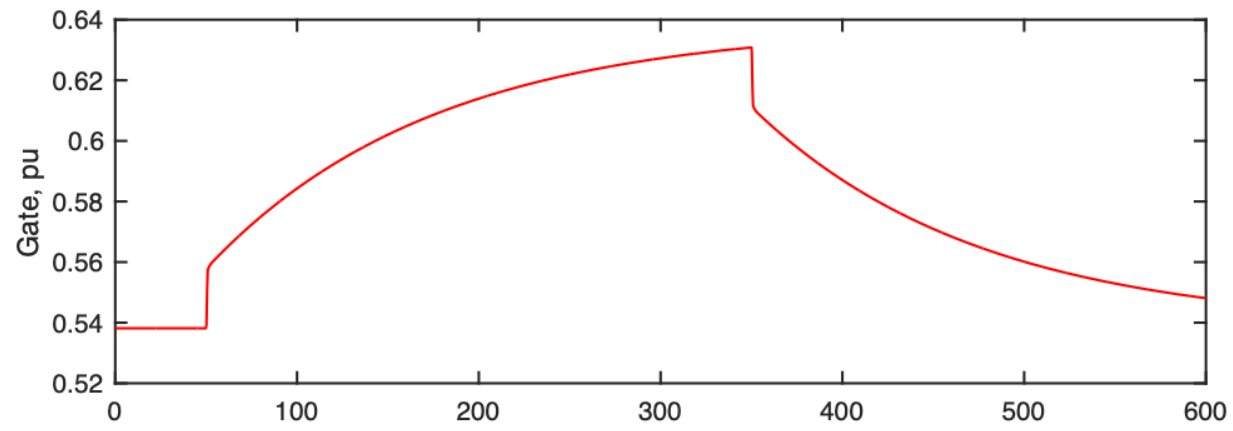
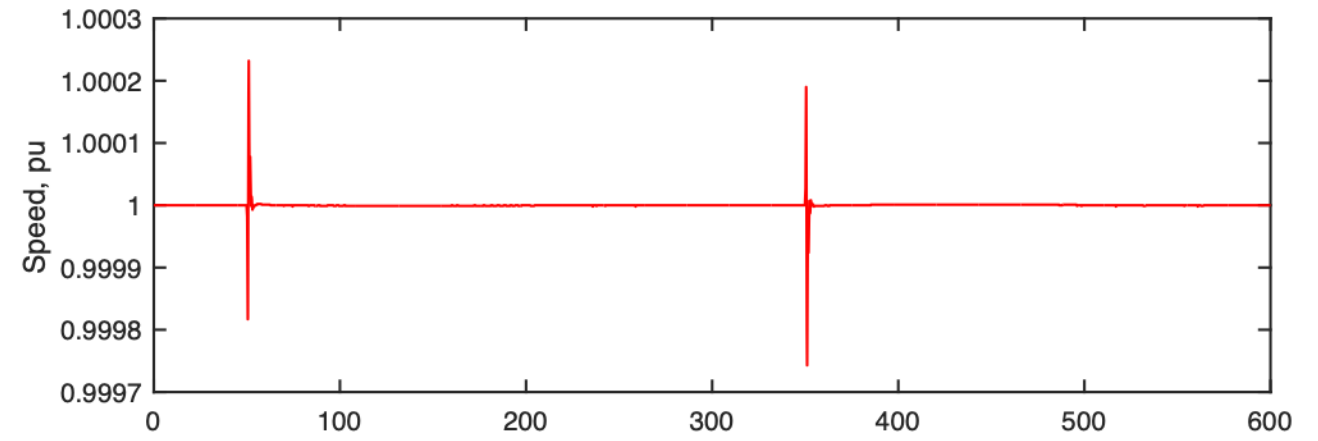
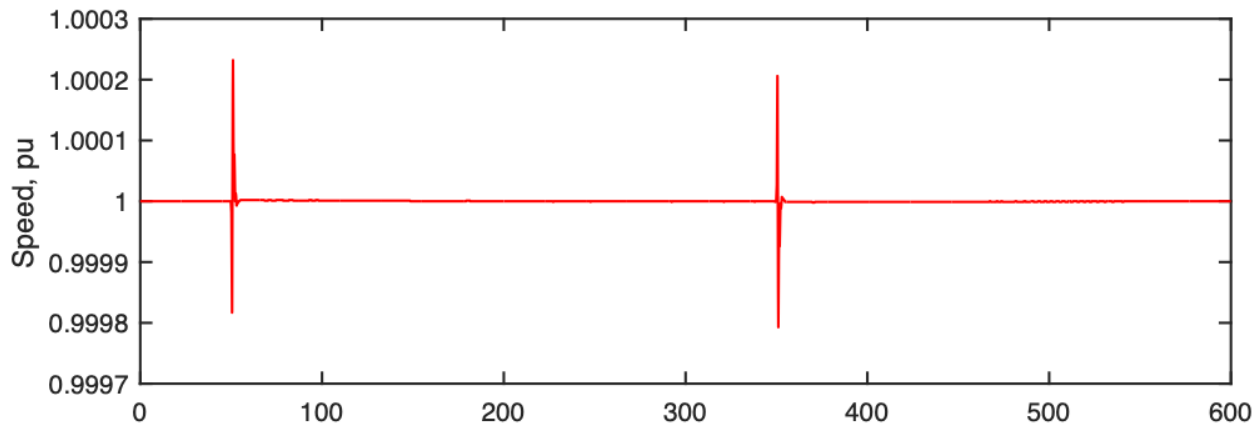
Governor ref step +0.005 %
Governor ref step -0.005 %

Red/Blue - test recording
Black - simulation (h6e)



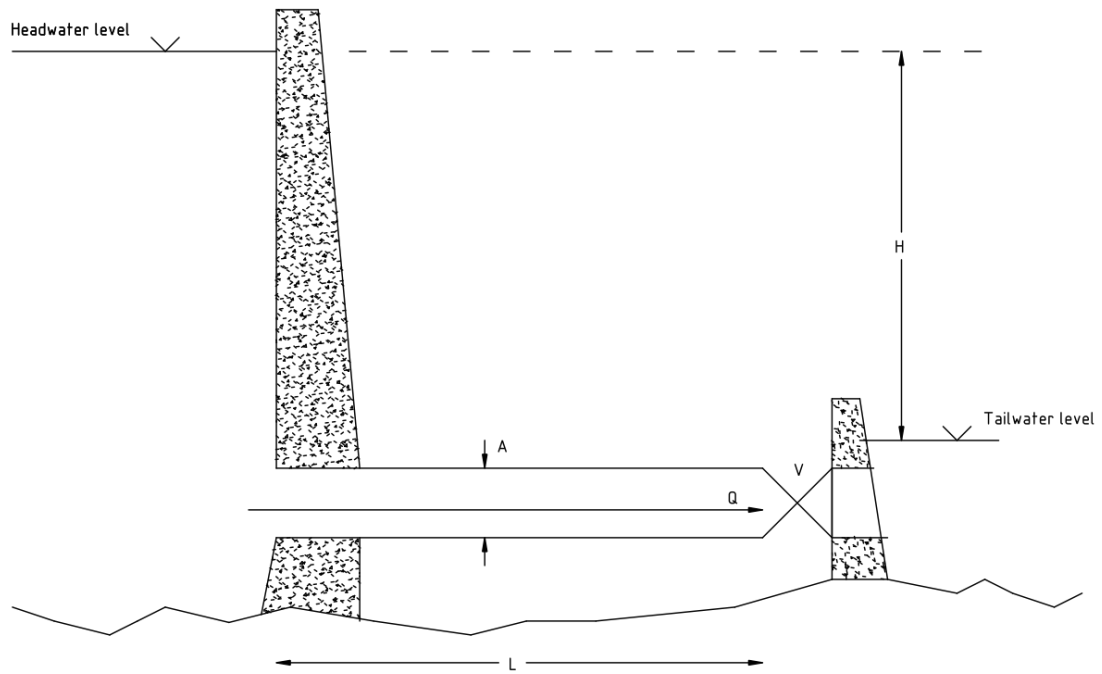
Load controller (generic model)



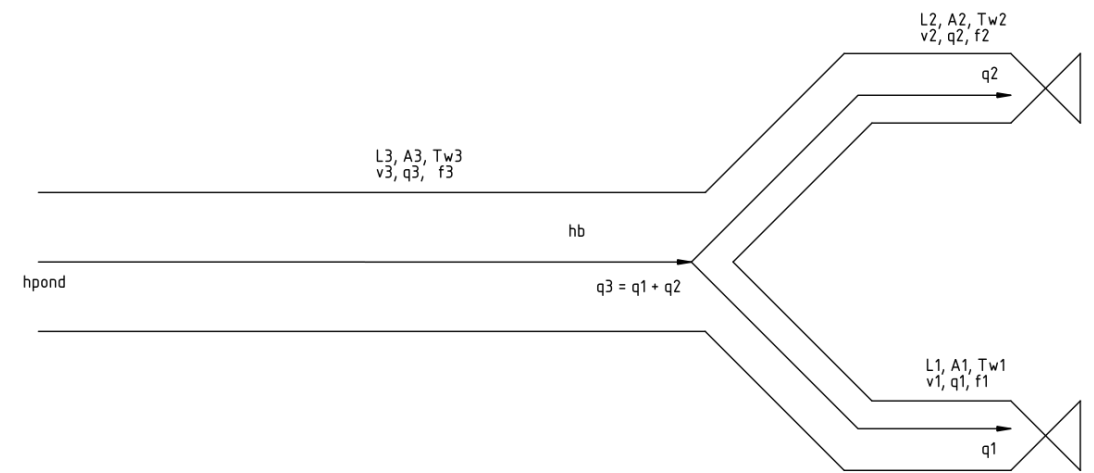


Governor-only

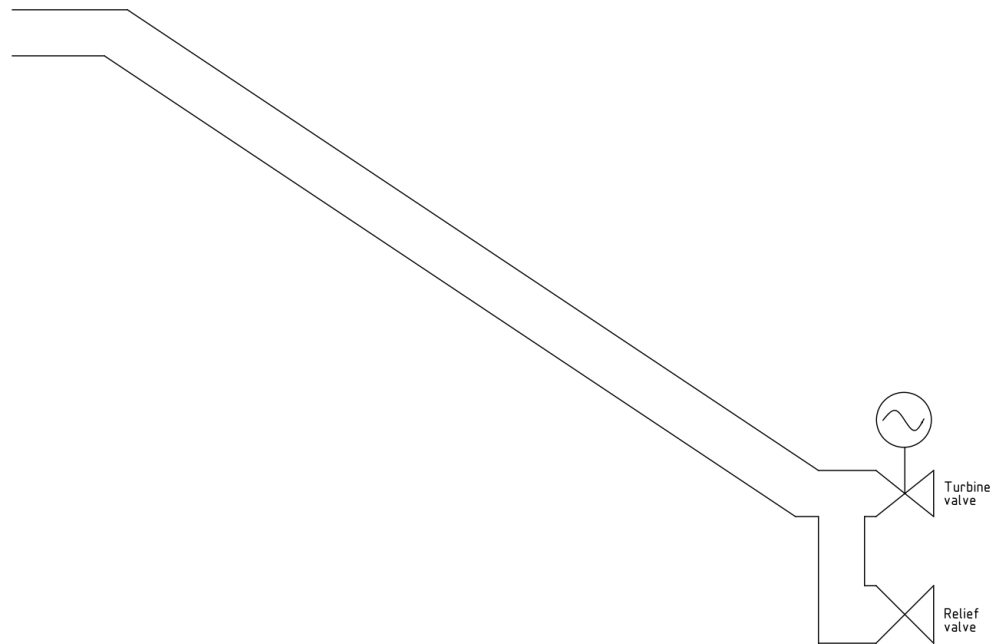
Load controller active



Straight forward penstock



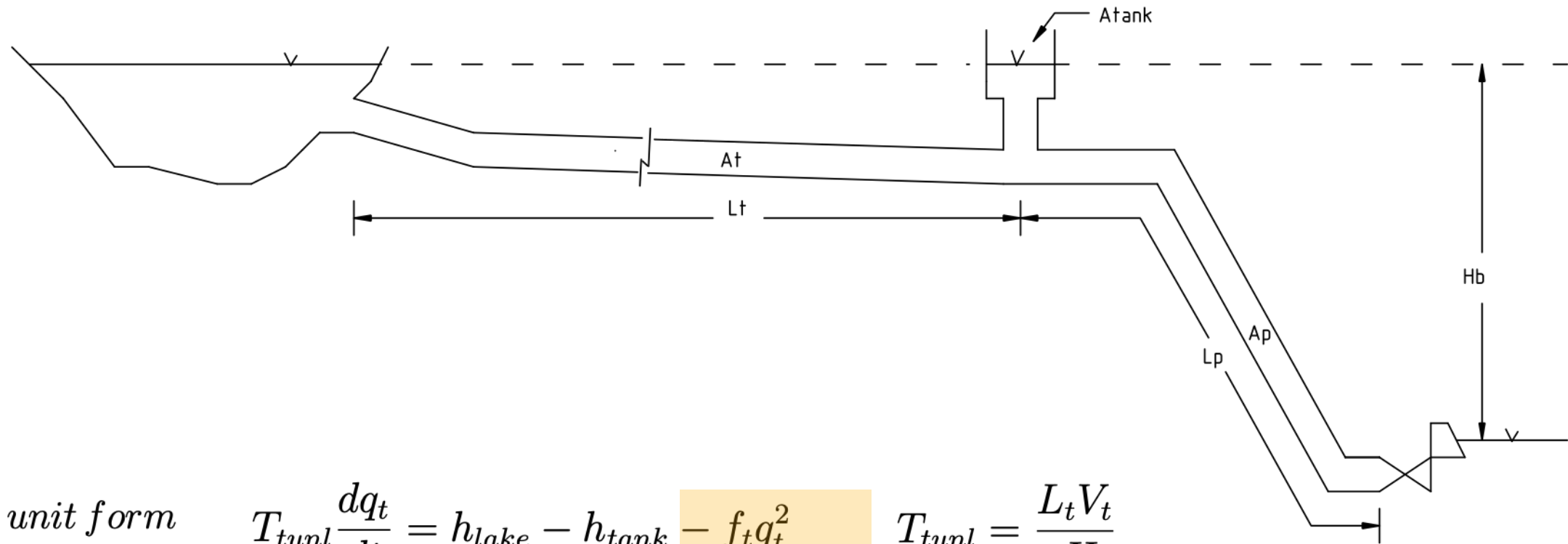
Common bifurcated penstock



Long penstock and relief valve



Tunnel/Surge Tank



Per unit form

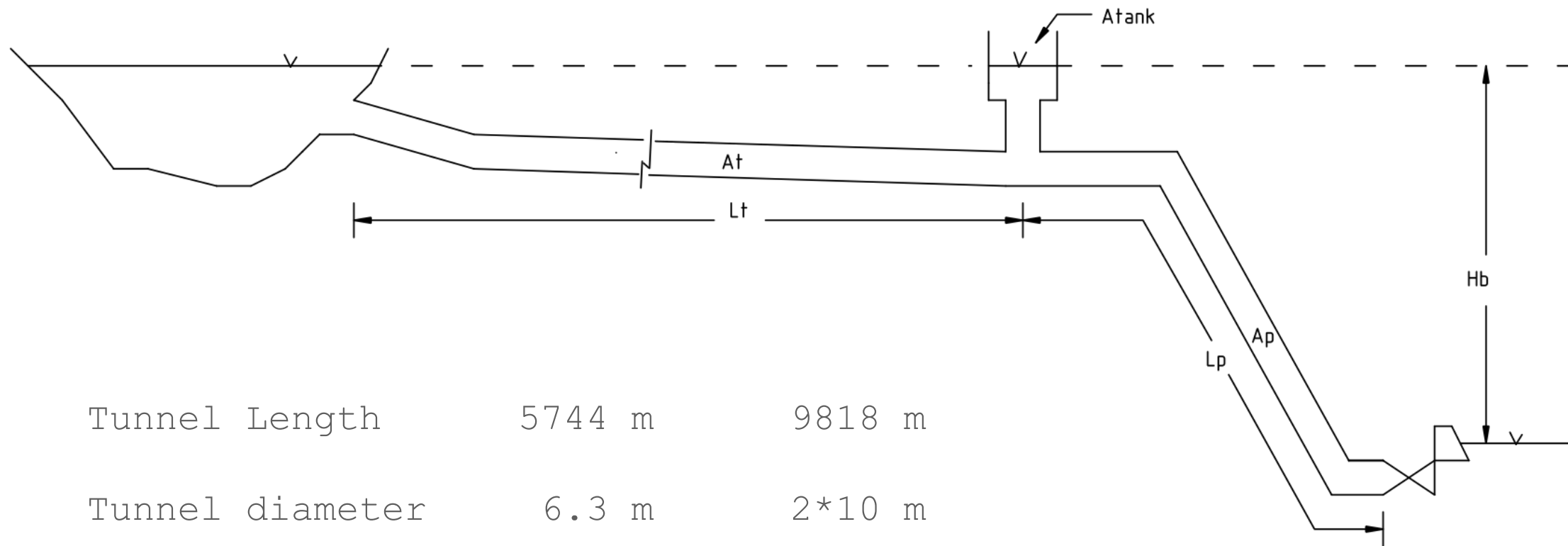
$$T_{tunl} \frac{dq_t}{dt} = h_{lake} - h_{tank} - f_t q_t^2 \quad T_{tunl} = \frac{L_t V_t}{g H_b}$$

$$T_{tpen} \frac{dq_p}{dt} = h_{tank} - h_{tail} - f_p q_p^2 \quad T_{tpen} = \frac{L_p V_p}{g H_b}$$

$$T_{tank} \frac{dh_{tank}}{dt} = q_t - q_p \quad T_{tank} = \frac{H_b}{A_{tank} Q_b}$$

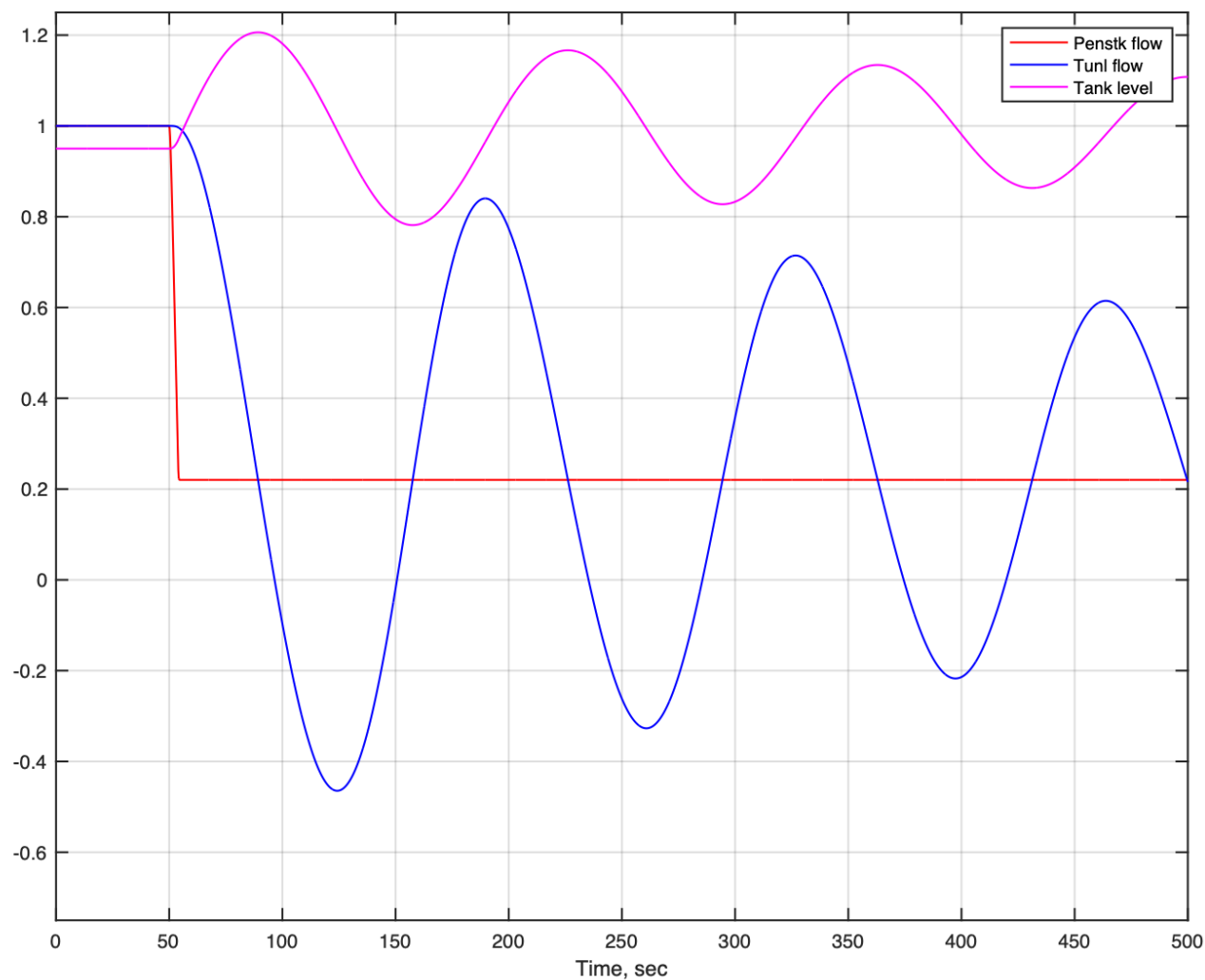
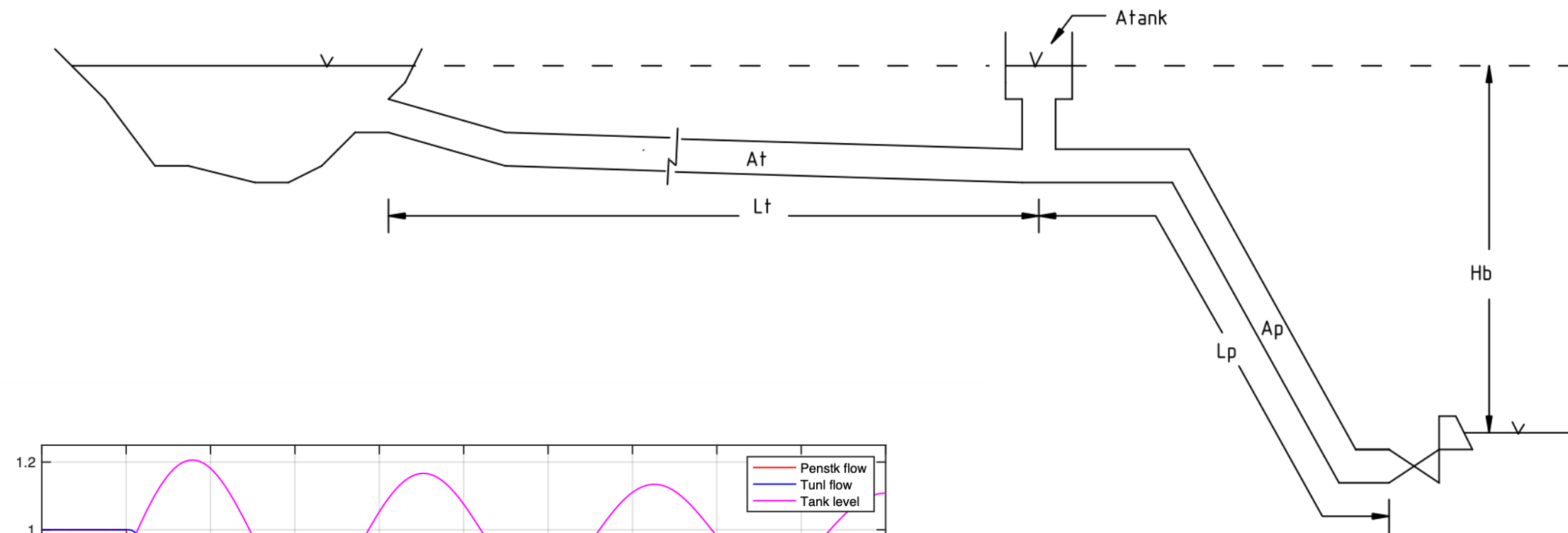
$F = \text{dimensionless loss coef}$

Tunnel/Surge Tank



Tunnel Length	5744 m	9818 m
Tunnel diameter	6.3 m	2*10 m
Surge tank dia	11.9 m	
Surge tank area	444 m ²	20,000 m ²
Base head	206.0 m	166 m
Penstock length	854 m	170 m
Nameplate power	4X50 MW	7*120 MW
Normal max flow	99 cms	425 cms

Tunnel/Surge Tank



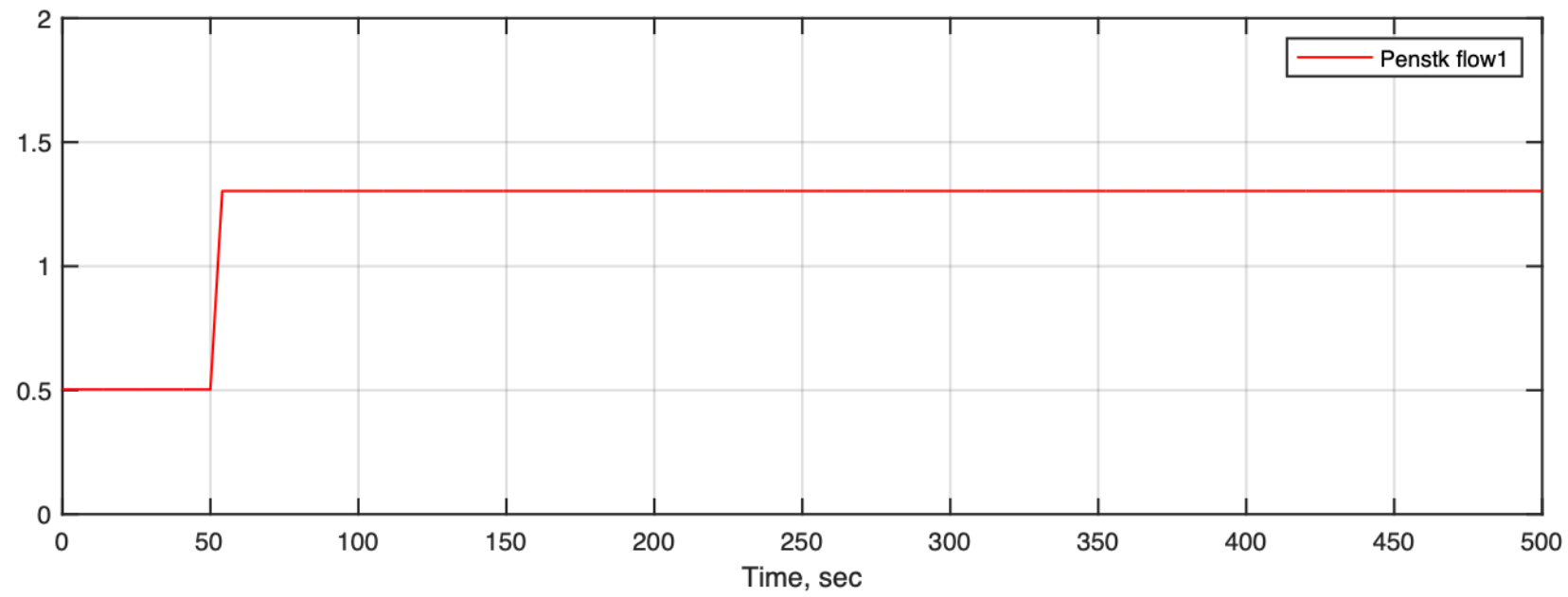
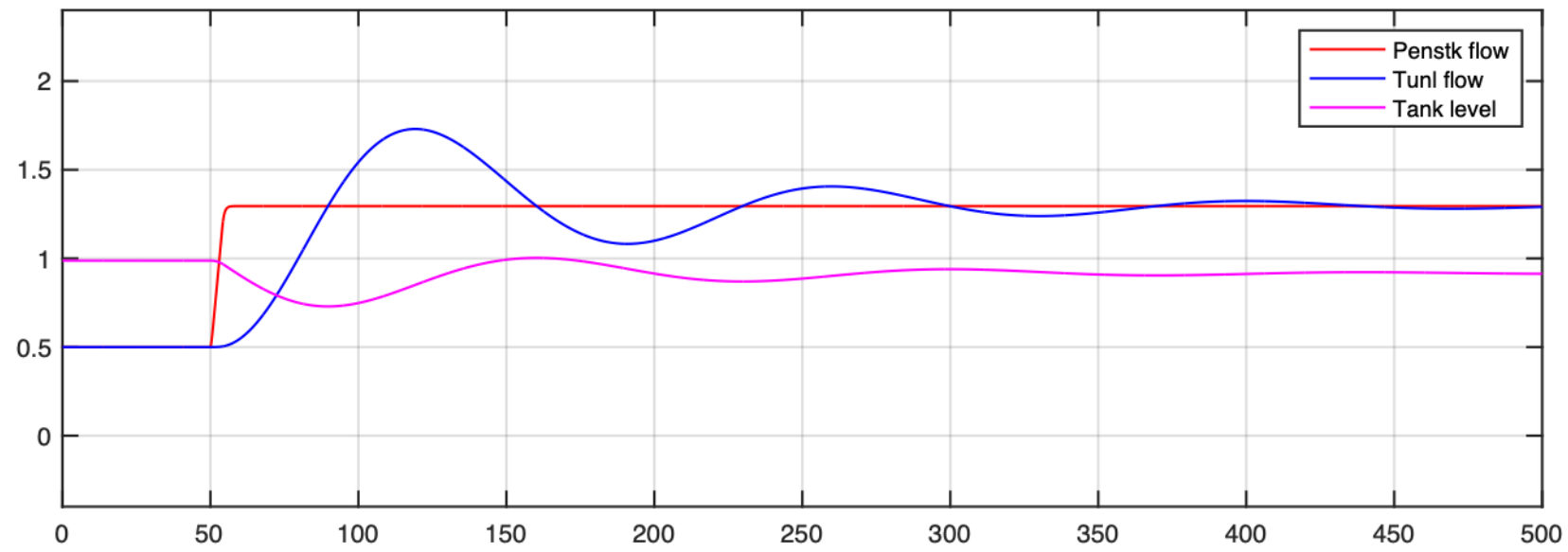
$L_t = 2120 \text{ m}$
 $L_p = 200 \text{ m}$
 $H_b = 152 \text{ m}$
 $f_t = 0.05$
 $T_{tunl} = 6.7 \text{ sec}$
 $T_{pen} = 1.0 \text{ sec}$
 $T_{tank} = 182 \text{ sec}$

Time scale of turbine/generator dynamics is very much shorter than time scale of tunnel flow transients.

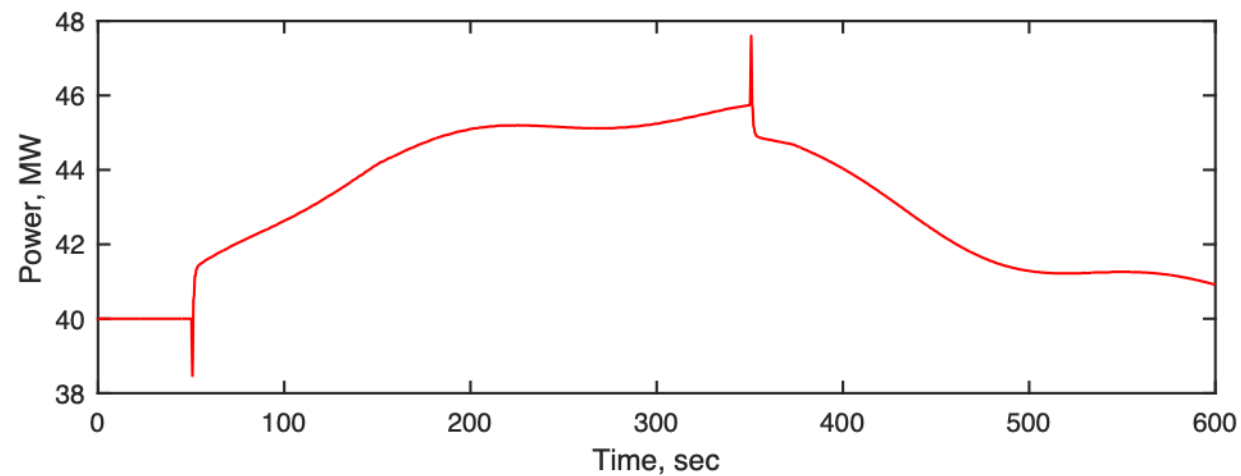
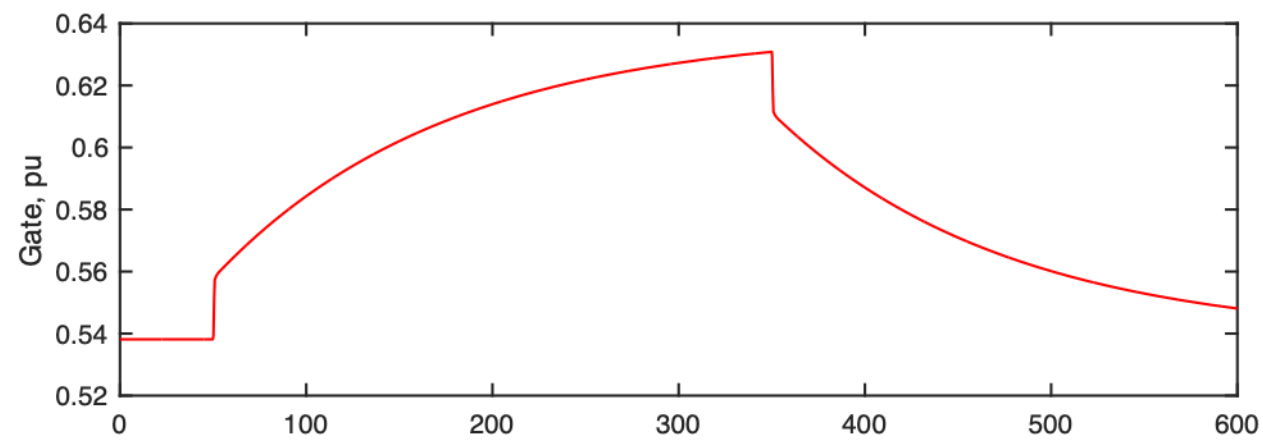
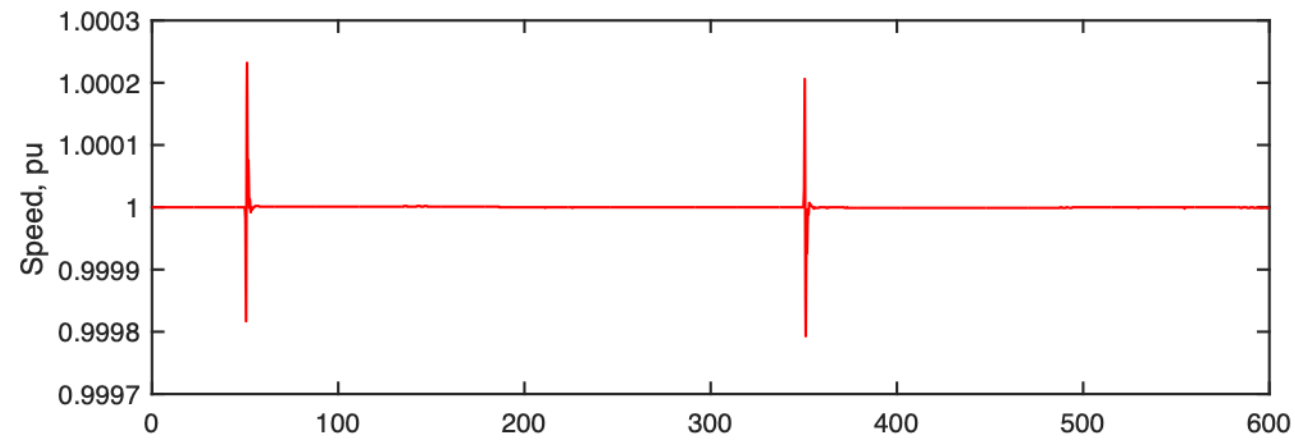
Quick change in penstock flow requires tunnel flow to divert into surge tank. Rise in surge tank surface level creates decelerating head to reduce tunnel flow

Tunnel oscillation periods are a few to many minutes

Head loss due to tunnel wall friction may be significant



Surge Tank A



On line

Proportional Gain 5.0

Integral gain 0.2

Penstock T_w 2.0

Surge tank T_{tank} 120

Tunnel T_{wt} 6.4

Tunnel friction coeff 0.01

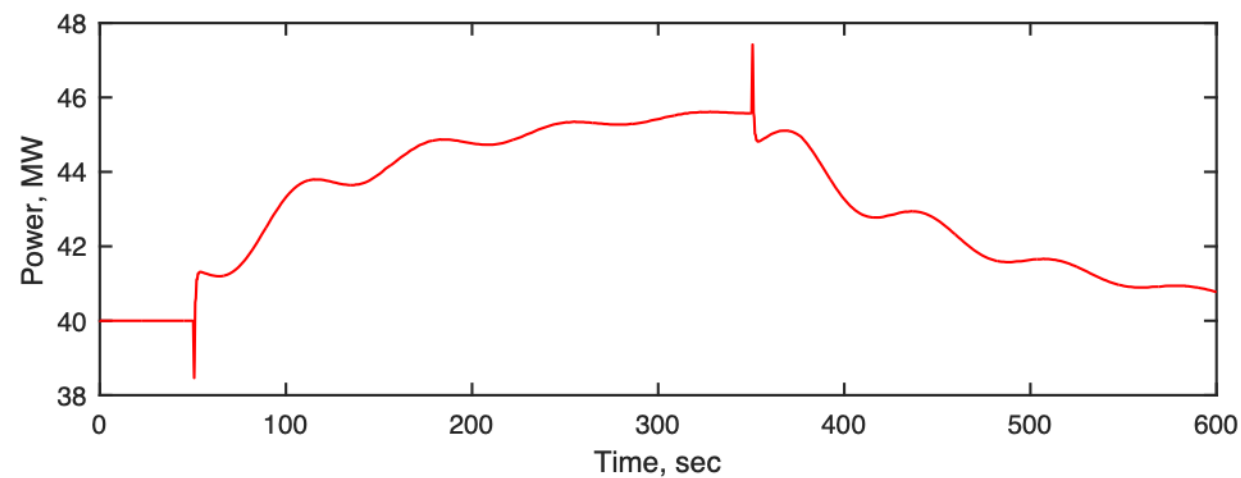
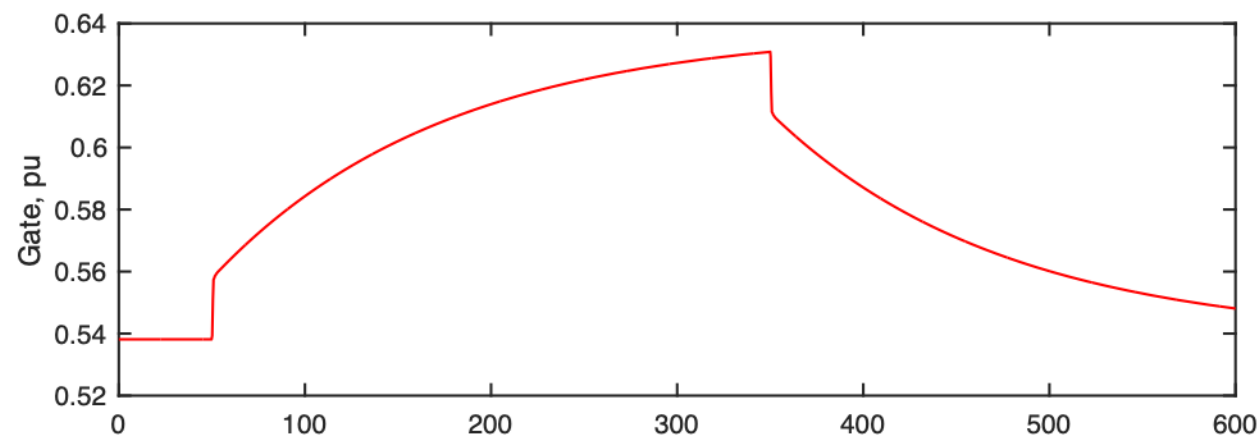
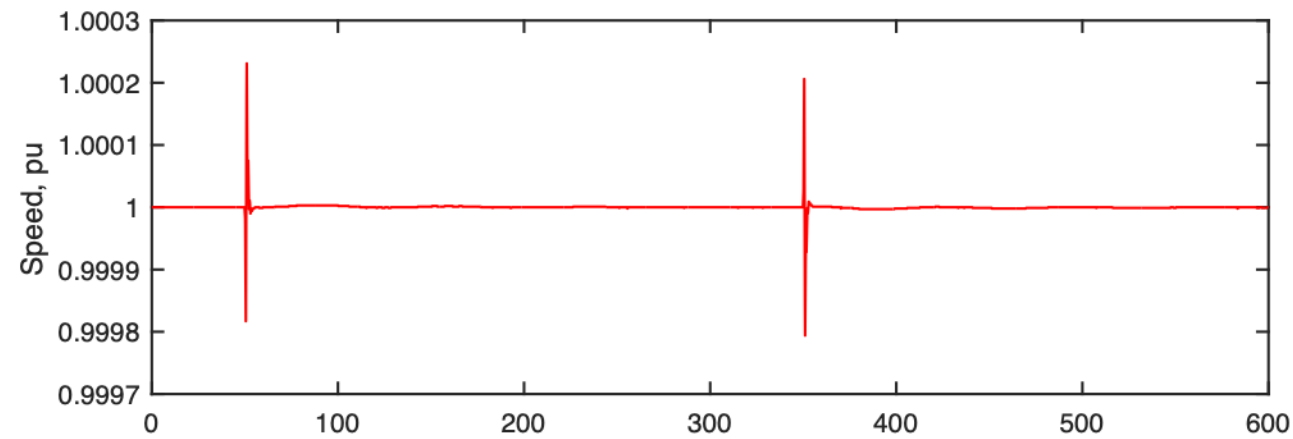
Initial output 40 MW

Load step 4 MW

Governor ref step 0.0 %

Generator on line, speed substantially fixed.
Governor proportional gain set to give
favorable response to speed adjustment.

Water supply by 2km tunnel. Amply sized surge
tank. Some reduction in early rate of output
increase, but change is small



On line

Proportional Gain 5.0

Integral gain 0.2

Penstock T_w 2.0

Surge tank T_{tank} 20

Tunnel T_{wt} 6.4

Tunnel friction coeff 0.01

Initial output 40 MW

Load step 4 MW

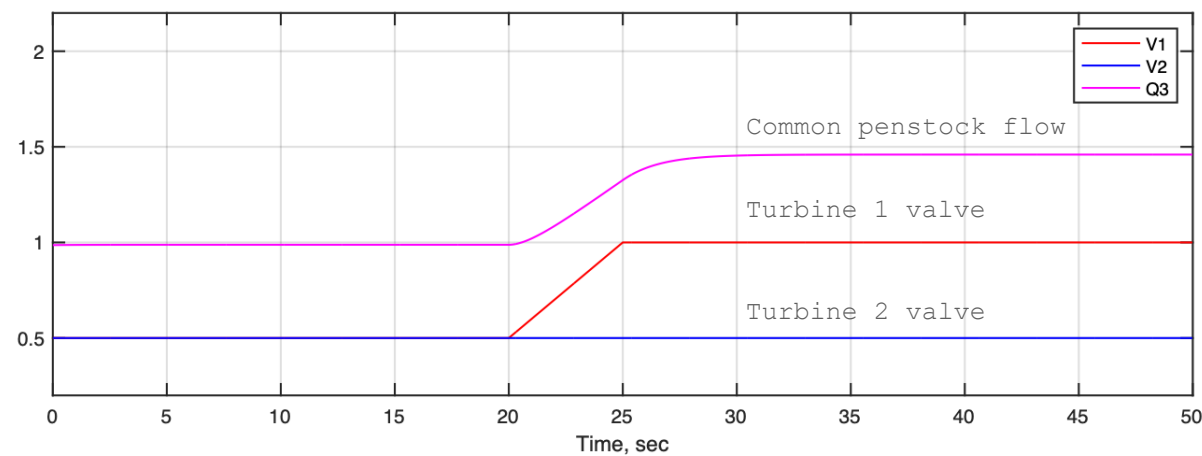
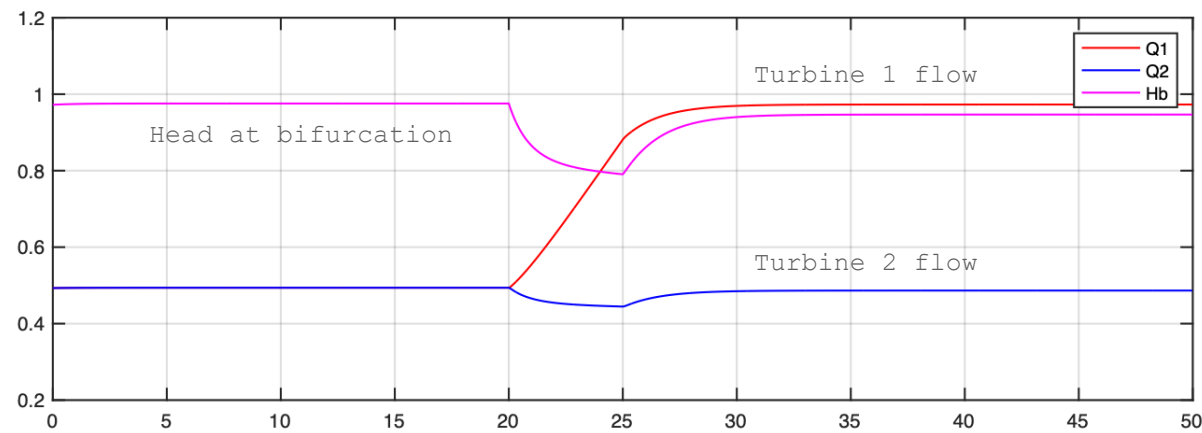
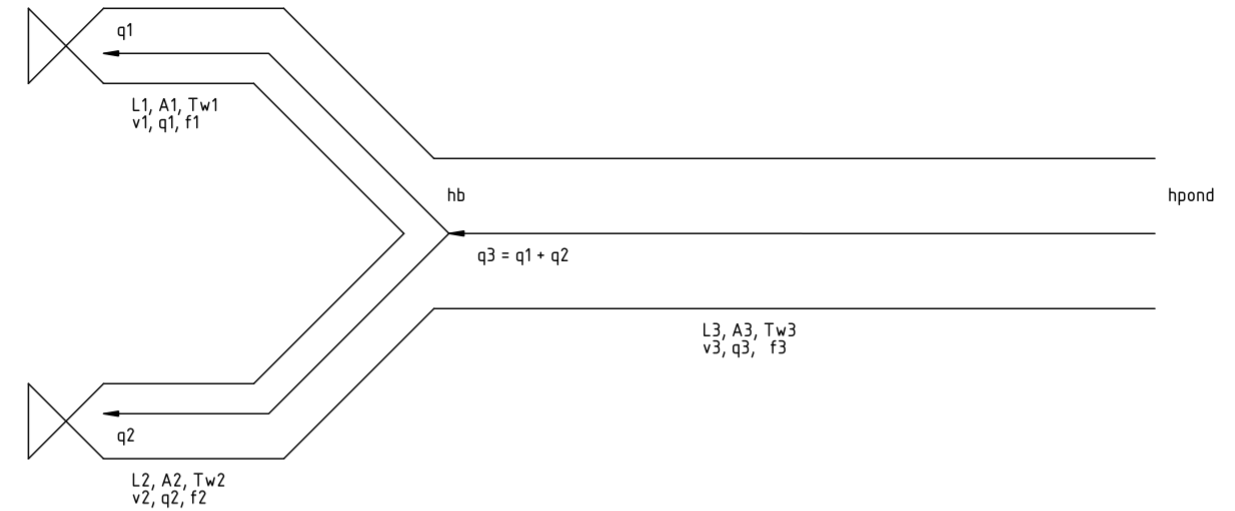
Governor ref step 0.0 %

Generator on line, speed substantially fixed. Governor proportional gain set to give favorable response to speed adjustment.

Water supply by 2km tunnel. Undersized surge tank. Ineffective in decoupling turbine and tunnel dynamic response. Strong oscillation at characteristic frequency of tunnel/surge tank subsystem. Significant reduction in early rate of output increase

Bifurcated penstock

Inertial behavior of the common water mass transiently affects the division of flow between the two branches



Bifurcation

Common penstock time constant, T_{w3} 2.0 sec

Individual penstock time const, T_{w1}, T_{w2} 0.2 sec

Common loss coefficient, f_3 0.025

$$T_{w1} \frac{dq_1}{dt} = h_b - h_1 - f_1 q_1^2$$

$$T_{w2} \frac{dq_2}{dt} = h_b - h_2 - f_2 q_2^2$$

$$\frac{dq_1}{dt} + \frac{dq_2}{dt} = \frac{h_{pond} - h_b - f_3 (q_1 + q_2)^2}{T_{w3}}$$

Summing up

Many of the models available in PSS/E, PSLF, PW, etc were developed when computing limitations were much more restrictive than they are now - they do not address present operating concerns

Dependence of turbine capability on head	Seasonal to hourly
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Dispatch to meet flow/wildlife issues	Seasonal 24/7
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Dispatch of interrelated rivers/reservoirs	Seasonal to hourly
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Individual penstock/tunnel characteristics	Affects testing (mod026)
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...

Obsolete models can be deleted and new models can be introduced

Our study practices and, by association, our modeling of plant operations have lagged badly

Our main failing has not been in the modeling of plant dynamics

It has been in the managing of the 'dispatch' of hydro plant output in the LOAD FLOW base cases

Updating of turbine/governor/plant dynamic modeling should be done

but

it will be of limited value if it is not accompanied by improvement of data and practices used in dispatching hydro generation

■ ■ ■

Hydro plants are like human beings:

In many respects 'they are all the same'

In key respects 'every one is different'

'day is different'

Adding detail to a model may be necessary to deal with a particular issue but may not improve the accuracy with regard to issues of primary concern in interconnection studies

Data base design and associated design of simulation models must be done with care as to when:

One 'size fits all' modeling is sufficient

Particular models are needed for particular study purposes



Modeling used in data validation tests (mod026) may call for greater detail than is essential for interconnected-grid studies

- turbine flow/power relationship at-and-near FSNL
- shared penstock detail

Modeling used for hydraulic operational issues requires representation of water path that is valid for much longer periods than are considered in grid-wide studies

- tunnel/canal/pondage details
- relief valve details
- draft tube vortex and rough running

Modeling for isolated load / black start operation requires details of

- changing governor modes and gain settings
- timing characteristics of power transducers, gate position feedback

The dilemma of detail

Detail is not an assurance of accuracy

Adding detail to a model may be needed when dealing with a particular issue

But adding detail may not improve the accuracy of the model with regard to other issues

and

can give a false impression of accuracy to unwary users and observers

Conclusion

Our main failing has not been in the modeling of plant dynamics

It has been in the managing of hydro plant dispatch in power flow base cases

Updating of turbine/governor/plant dynamic modeling should be done

but

the important requirement is to improve the data and practices for dispatching hydro generation in study base cases

Thank you